Discrete Vortex Modeling of Inviscid Flow in Aerodynamic Flutter

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Overview

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  • Discrete Vortex Method
  • Dynamic Motion Equation
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Introduction

Aerodynamic flutter is the unstable oscillation of a body caused by the interaction of aerodynamic forces, structural elasticity and inertial effects.
Introduction: Objective

- To model a simple case of aerodynamic flutter through the use of discrete vortex method (DVM)
  - Much faster than FEA approach

- Assumptions
  - Inviscid flow (mimics highly turbulent flow)
  - Impulsively-started flow
Inviscid Flow Theory: Complex Potential

• Neglect boundary layer (no viscosity)
• Potential flow
  • Describe velocity field as a gradient of the velocity potential
    \[ V = \nabla \phi \]
• Complex potential
  • Describe position of points with complex numbers
    \[ z = x + iy \]
    \[ w(z) = \phi + i\psi \]
Inviscid Flow Theory: Complex Potential

- Elementary Flow Types

*Uniform*  *Source*  *Vortex*  *Doublet*
Inviscid Flow Theory: Complex Potential

• Superposition of elementary flow types

\[ w(z) = U(z) \quad w(z) = U\left(\frac{a^2}{z}\right) \quad w(z) = U\left(z + \frac{a^2}{z}\right) \]
Inviscid Flow Theory: Conformal Mapping

- 2D potential flow can be transformed to another complex plane

\[ z = x + iy, \quad w = \phi + i\psi \]

\[ f(z) = w \]

\[ \frac{dw}{dz} = u - iv \]
Inviscid Flow Theory: Conformal Mapping
Discrete Vortex Method

• Represent a continuously distributed vorticity with many elemental discrete line vortices
• 1979: Sarpkaya modeled flow past a cylinder

\[
w(z_1) = U \left( z_1 + \frac{a^2}{z_1} \right) - \frac{i}{2\pi} \sum_{j=1}^{m} \Gamma_j \left[ \ln(z_1 - z_{1,j}) - \ln \left( z_1 - \frac{a^2}{z_{1,j}} \right) \right]
\]
Discrete Vortex Method

• Nascent vortices added to the flow field with each time step
  • Release distance:
    \[ z_n = \left( 1 + \frac{|\Gamma_n|}{2\pi U_s} \right) \left( 1 - \frac{|\Gamma_n|}{2\pi U_s} \right) e^{i(\pi - \theta_s)} \]
  • Circulation strength:
    \[ \Gamma_n = \frac{1}{2} U_s^2 \cdot dt \]
  • All vortices are transported downstream
    \[ z_{t + dt} = z_t + v \cdot dt \]
Dynamic Motion Equation

• Aerodynamic forces create a moment

\[ C_p = 1 - \frac{V^2}{U^2} \]

\[ F_x(r) = C_p(r) \sin \theta, \quad F_y(r) = C_p(r) \cos \theta \]

\[ M_z = \int_{-2a}^{2a} (xF_y - yF_x) \, dr \]
Dynamic Motion Equation

• Flutter is simplified to torsion about its center

\[ I_{zz} \ddot{\theta} + b \dot{\theta} + \kappa \theta = M_z(t) \]

• Second and first derivatives approximated by finite differencing

\[
\ddot{\theta} \approx \frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{(\Delta t)^2}, \quad \dot{\theta} \approx \frac{\theta_{i+1} - \theta_i}{\Delta t}
\]
Methods

• FreeBasic
  • Compiled language
  • Built-in graphics
  • Very fast
  • Free!
Methods

• Input
  • Flow properties
  • Plate properties
  • Display preferences

• Output
  • Displays inviscid streamlines and positions of all vortices in z-planes
  • Pressure and force distribution plots
  • Deflection angle, drag force, and pressure probe data at each time step
Methods
Results: Inviscid Flow

Stationary Plate at varying Deflection Angles, z6-plane

- 0°
- 30°
- 60°
Results: Inviscid Flow

Stationary Plate at $0^\circ$, pressure distribution

Pressure Distribution
Results: Inviscid Flow

Oscillating Plate, z-6 plane (left) and pressure distribution (right)
Discussion: Inviscid Flow

- Net force = 0
- Highest force at stagnation points ($C_p = 1$)
- Negative force at regions of high velocity
Results: Discrete Vortices

Stationary Plate at 0°, z6-plane

<table>
<thead>
<tr>
<th>Plane</th>
<th>Deflection Angle</th>
<th>Initial Angle</th>
<th>Number of Vortices</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>z6</td>
<td>0 (deg)</td>
<td>0 (deg)</td>
<td>20</td>
<td>-0.125</td>
</tr>
</tbody>
</table>

Speed x10
Discussion: Discrete Vortices

• Von Kármán vortex street
• Total circulation = 0 (Kelvin Circulation Theorem)
Results: Discrete Vortices

Stationary Plate at 0°, z6-plane (left) and force distribution (right)
Results: Discrete Vortices

Stationary Plate at 0°
Results: Discrete Vortices

Stationary Plate at 0°
Results: Flutter

Flutter simulation, \(z_6\)-plane (left) and force distribution (right)

Speed x1
Results: Flutter

Comparing different torsional stiffness values, less rigid (left) and more rigid(right).
Results: Flutter

Comparing different torsional rigidity values
Results: Flutter

Comparing different torsional rigidity values
Discussion: Flutter

- Second order differential equation (unstable)
- Higher torsional rigidity corresponds to smaller deflections and longer time until failure
Conclusion

• DVM provides an efficient method of simulating a simple case of aerodynamic flutter

• The results appear realistic and match the results of previous work and predictions
Conclusion: Further Work

- Different approaches to singularities in inviscid flow
  - Free surface theory
  - Sarpkaya’s averaging technique
- Examine wake shape
- Fast Fourier Transform on pressure probe data: $St = f(Re)$
- Comparisons with experimental data
- Fatigue considerations
- Conformal mapping to other profiles
  - Streamlined strut
  - Joukowski airfoil
    - Thwaite’s Method to find separation points

Source: wolfram.com
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Questions?

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