Aerospace Systems and Technology Conference
American Institute of Aeronautics and Astronautics: Orange County Section
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Discrete Vortex Modeling of Inviscid Flow in Aerodynamic Flutter

Presented by:

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Overview

- Introduction
- Theory
  - Inviscid Flow Theory
  - Discrete Vortex Method
  - Dynamic Motion Equation
- Methods
- Results & Discussion
- Conclusion
  - Further Work

Introduction

Aerodynamic flutter is the unstable oscillation of a body caused by the interaction of aerodynamic forces, structural elasticity and inertial effects.

Source: youtube.com

Source: youtube.com

Source: youtube.com

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Introduction: Objective

- To model a simple case of aerodynamic flutter through the use of discrete vortex method (DVM)
  - Much faster than FEA approach

- Assumptions
  - Inviscid flow (mimics highly turbulent flow)
  - Impulsively-started flow

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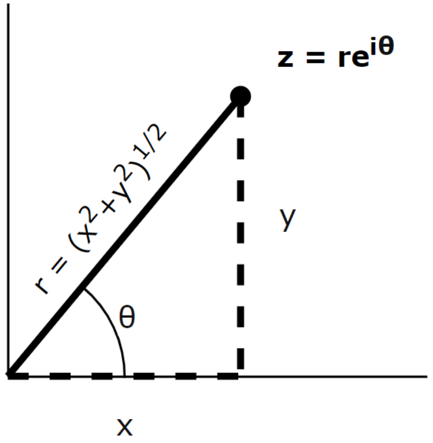
Inviscid Flow Theory: Complex Potential

- Neglect boundary layer (no viscosity)
- Potential flow
  - Describe velocity field as a gradient of the velocity potential

$$\mathrm{V} = \nabla\phi$$

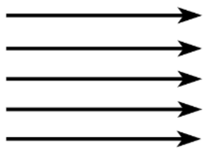


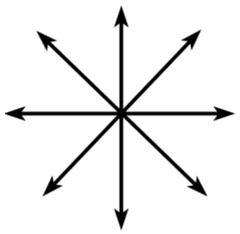
- Complex potential
  - Describe position of points with complex numbers

$$z = x + iy$$
$$w(z) = \phi + i\psi$$

Inviscid Flow Theory: Complex Potential

- Elementary Flow Types


Uniform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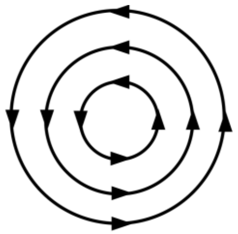
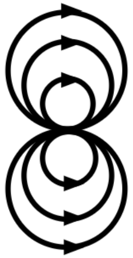
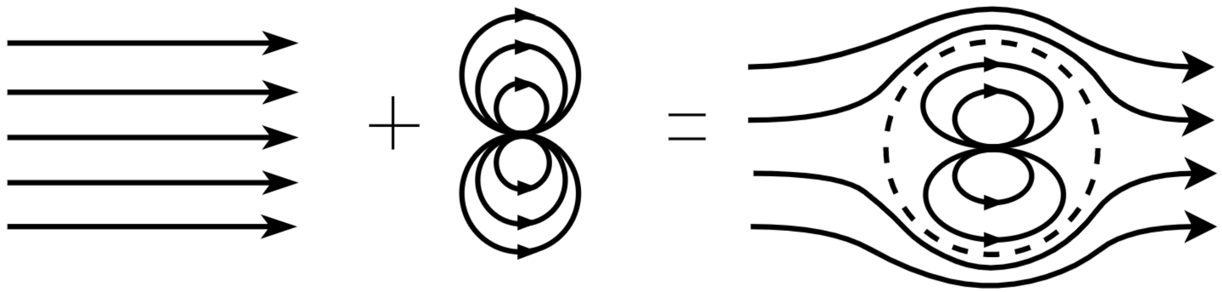

Vortex


Doublet

Inviscid Flow Theory: Complex Potential

- Superposition of elementary flow types

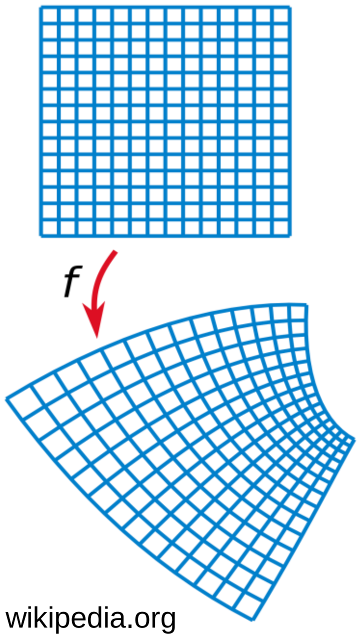


Uniform Flow | Doublet | Flow over a Cylinder

$$w(z) = U(z) \qquad w(z) = U\left(\frac{a^2}{z}\right) \qquad w(z) = U\left(z + \frac{a^2}{z}\right)$$

Inviscid Flow Theory: Conformal Mapping

- 2D potential flow can be transformed to another complex plane



Source: wikipedia.org

$$z = x + iy, \qquad w = \phi + i\psi$$

$$\boldsymbol{f(z) = w}$$

$$\boldsymbol{\frac{dw}{dz} = u - iv}$$

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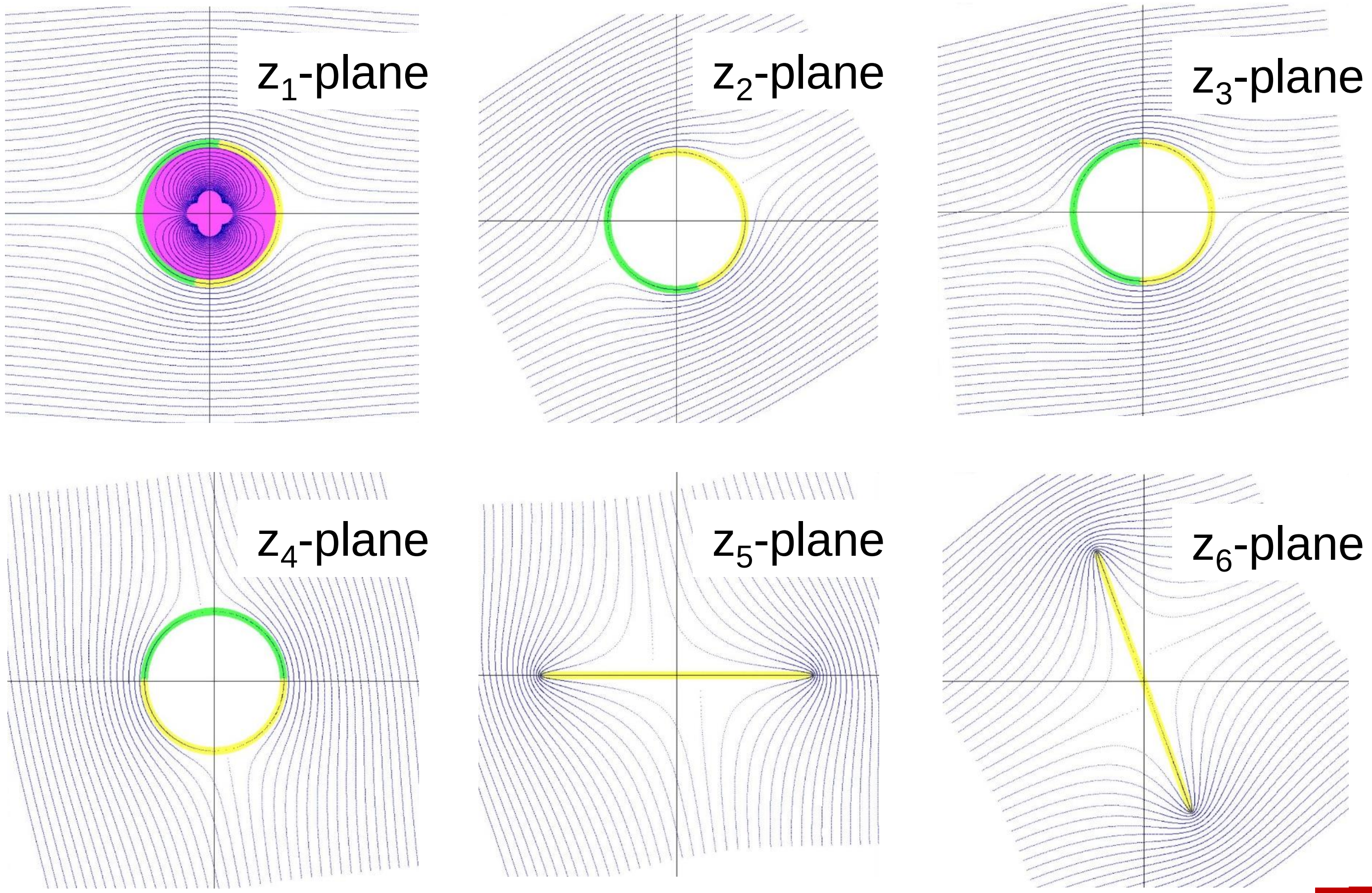
Inviscid Flow Theory: Conformal Mapping



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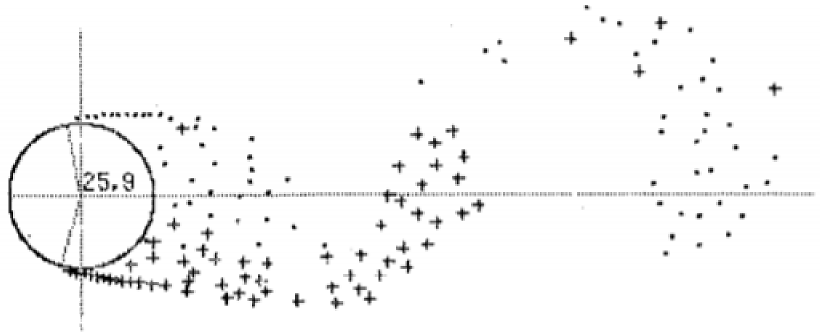
Discrete Vortex Method



Source: Sarpkaya

- Represent a continuously distributed vorticity with many elemental discrete line vortices
- 1979: Sarpkaya modeled flow past a cylinder

$$w(z_1) = U\left(z_1 + \frac{a^2}{z_1}\right) - \frac{i}{2\pi}\sum_{j=1}^{m}\Gamma_j\left[\ln\left(z_1 - z_{1,j}\right) - \ln\left(z_1 - \frac{a^2}{\overline{z}_{1,j}}\right)\right]$$

Discrete Vortex Method

- Nascent vortices added to the flow field with each time step
  - Release distance:

$$z_n = \left(\frac{1 + \frac{|\Gamma_n|}{2\pi U_s}}{1 - \frac{|\Gamma_n|}{2\pi U_s}} \right) e^{i(\pi - \theta_s)}$$

  - Circulation strength:

$$\Gamma_n = \frac{1}{2} U_s^2 * dt$$

- All vortices are transported downstream

$$z_{t+dt} = z_t + v * dt$$

Dynamic Motion Equation

- Aerodynamic forces create a moment

$$C_p = 1 - \frac{V^2}{U^2}$$

$$F_x(r) = C_p(r)\sin\theta\,, \qquad F_y(r) = C_p(r)\cos\theta$$

$$M_z = \int_{-2a}^{2a} (xF_y - yF_x)\,dr$$

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Dynamic Motion Equation

- Flutter is simplified to torsion about its center

$$I_{zz}\ddot{\theta} + b\dot{\theta} + \kappa\theta = M_z(t)$$

- Second and first derivatives approximated by finite differencing

Central Difference *Forward Difference*

$$\ddot{\theta} \cong \frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{(\Delta t)^2}, \qquad \dot{\theta} \cong \frac{\theta_{i+1} - \theta_i}{\Delta t}$$

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Methods

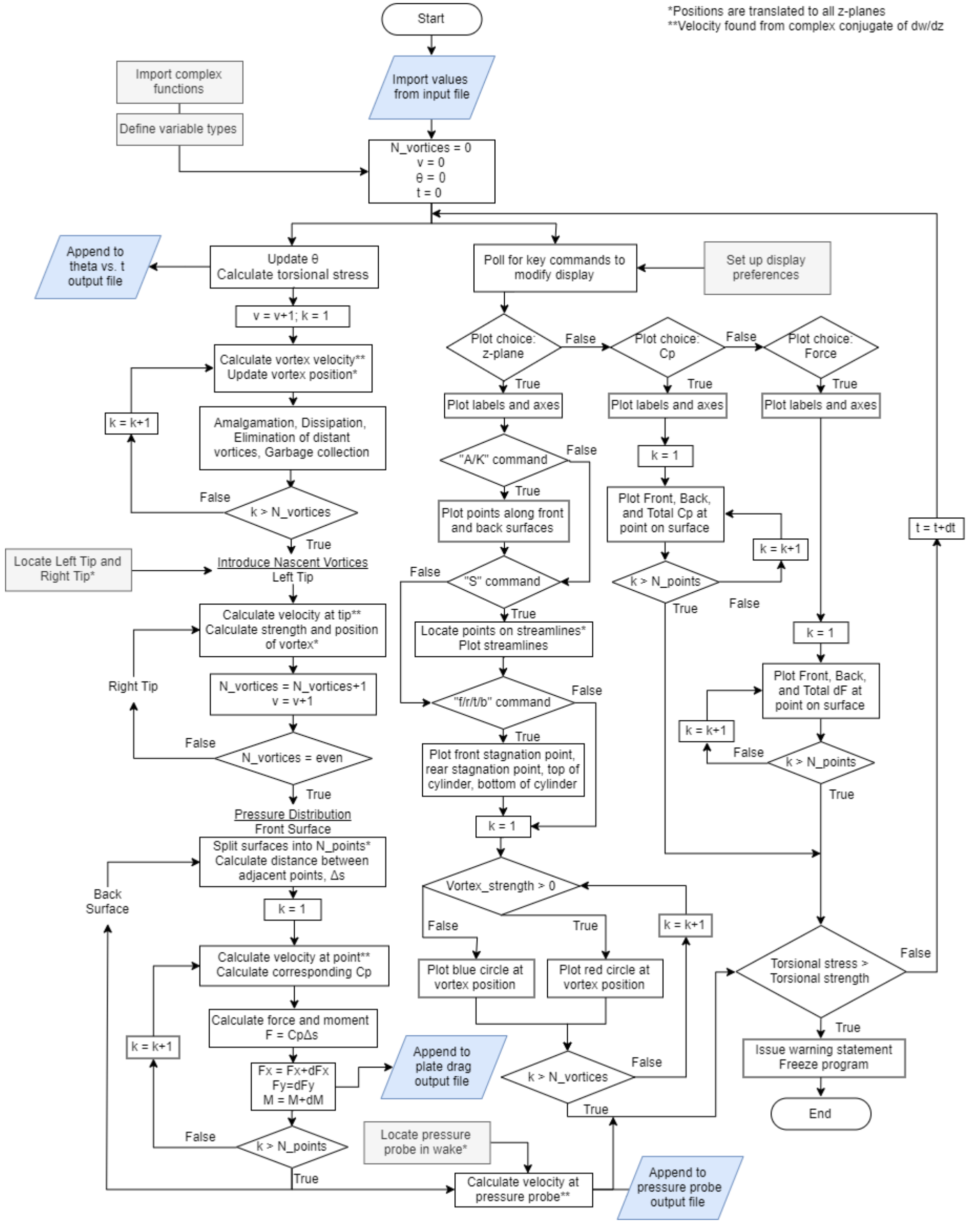
- FreeBasic
  - Compiled language
  - Built-in graphics
  - Very fast
  - Free!

Methods

- Input
  - Flow properties
  - Plate properties
  - Display preferences

- Output
  - Displays inviscid streamlines and positions of all vortices in z-planes
  - Pressure and force distribution plots
  - Deflection angle, drag force, and pressure probe data at each time step

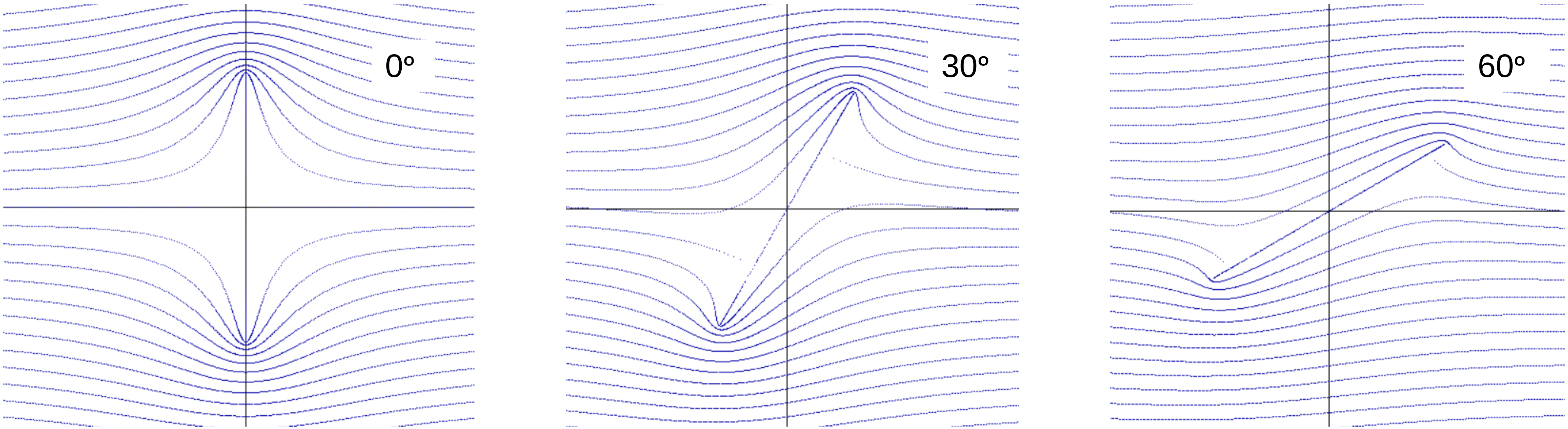
Methods



Start
*Positions are translated to all z-planes
**Velocity found from complex conjugate of dw/dz
Import complex functions
Define variable types
Import values from input file
N_vortices = 0
v = 0
θ = 0
t = 0
Append to theta vs. t output file
Update θ
Calculate torsional stress
v = v+1; k = 1
Calculate vortex velocity**
Update vortex position*
k = k+1
Amalgamation, Dissipation, Elimination of distant vortices, Garbage collection
False
k > N_vortices
True
Locate Left Tip and Right Tip*
Introduce Nascent Vortices
Left Tip
Calculate velocity at tip**
Calculate strength and position of vortex*
Right Tip
N_vortices = N_vortices+1
v = v+1
False
N_vortices = even
True
Pressure Distribution
Front Surface
Back Surface
Split surfaces into N_points*
Calculate distance between adjacent points, Δs
k = 1
Calculate velocity at point**
Calculate corresponding Cp
k = k+1
Calculate force and moment
F = CpΔs
Fx = Fx+dFx
Fy=dFy
M = M+dM
Append to plate drag output file
False
k > N_points
True
Locate pressure probe in wake*
Calculate velocity at pressure probe**
Append to pressure probe output file
Poll for key commands to modify display
Set up display preferences
Plot choice: z-plane
False
True
Plot labels and axes
"A/K" command
False
True
Plot points along front and back surfaces
"S" command
False
True
Locate points on streamlines*
Plot streamlines
"f/r/t/b" command
False
True
Plot front stagnation point, rear stagnation point, top of cylinder, bottom of cylinder
k = 1
Vortex_strength > 0
False
True
Plot blue circle at vortex position
Plot red circle at vortex position
k = k+1
k > N_vortices
False
True
Plot choice: Cp
False
True
Plot labels and axes
k = 1
Plot Front, Back, and Total Cp at point on surface
k = k+1
k > N_points
True
False
Plot choice: Force
True
Plot labels and axes
k = 1
Plot Front, Back, and Total dF at point on surface
k = k+1
False
k > N_points
True
t = t+dt
Torsional stress > Torsional strength
False
True
Issue warning statement
Freeze program
End

Results: Inviscid Flow

Stationary Plate at varying Deflection Angles, z6-plane



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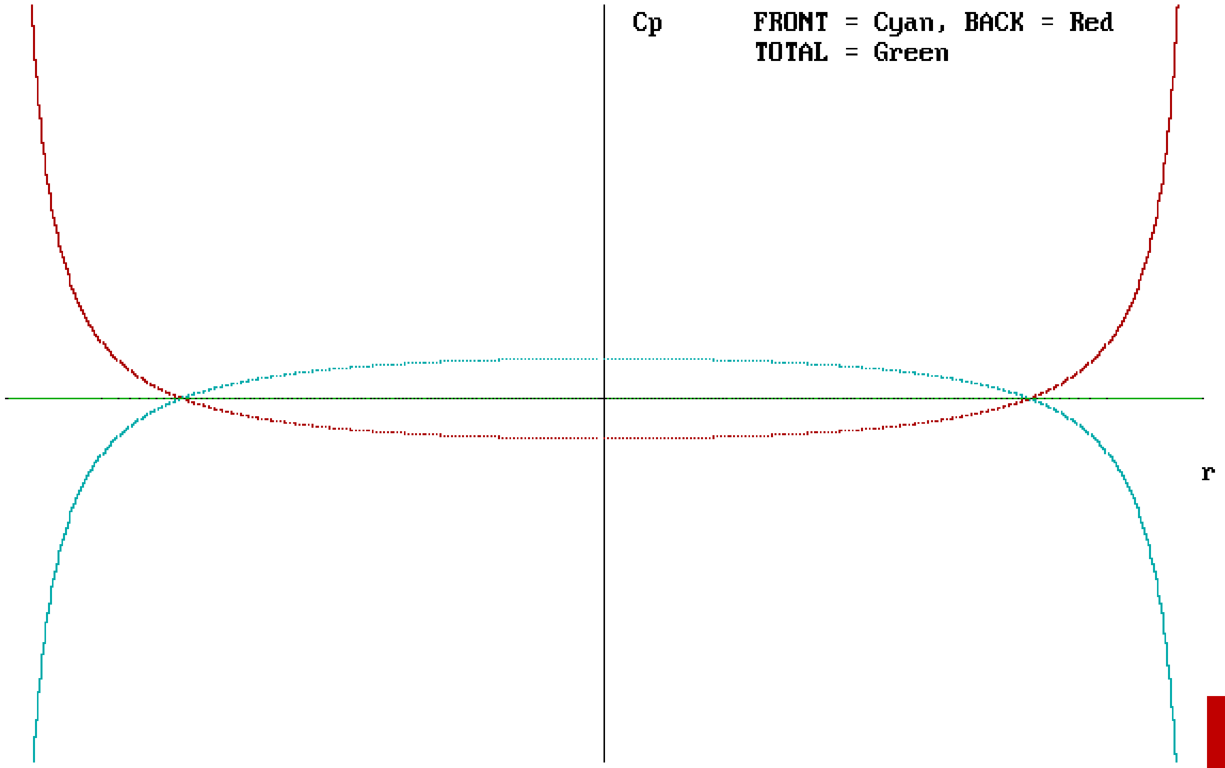
Results: Inviscid Flow

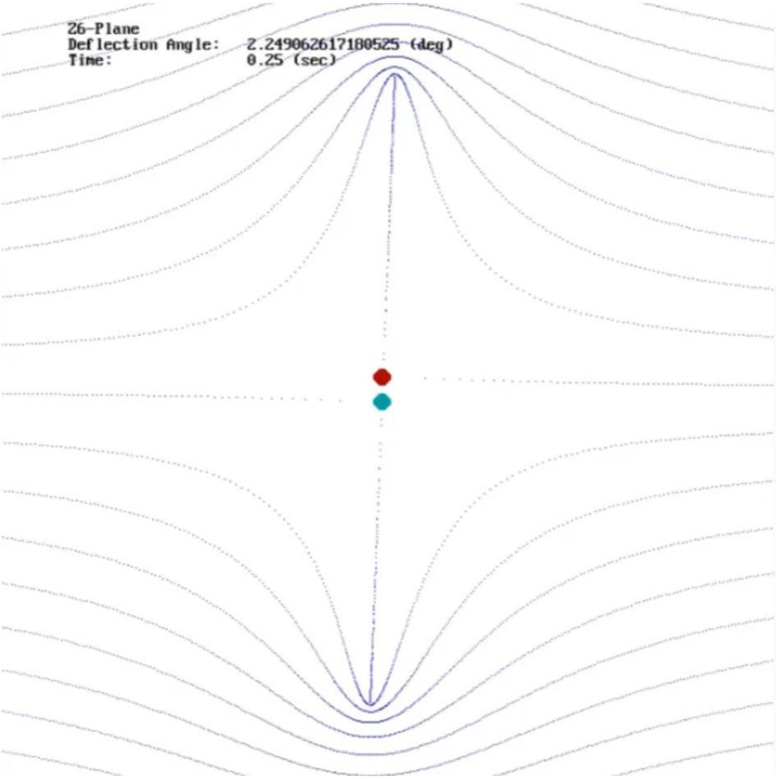
Stationary Plate at 0°, pressure distribution

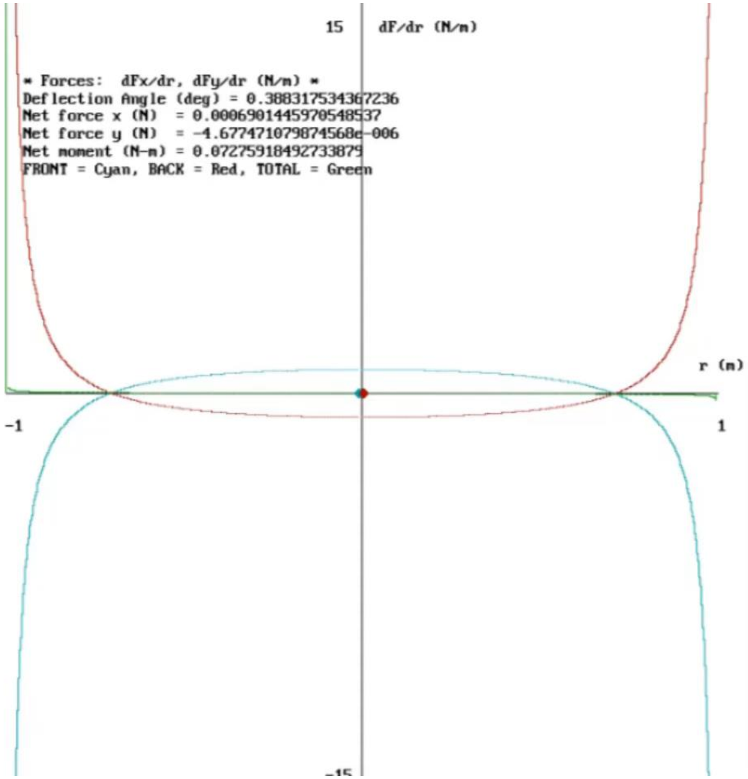
Pressure Distribution



Results: Inviscid Flow

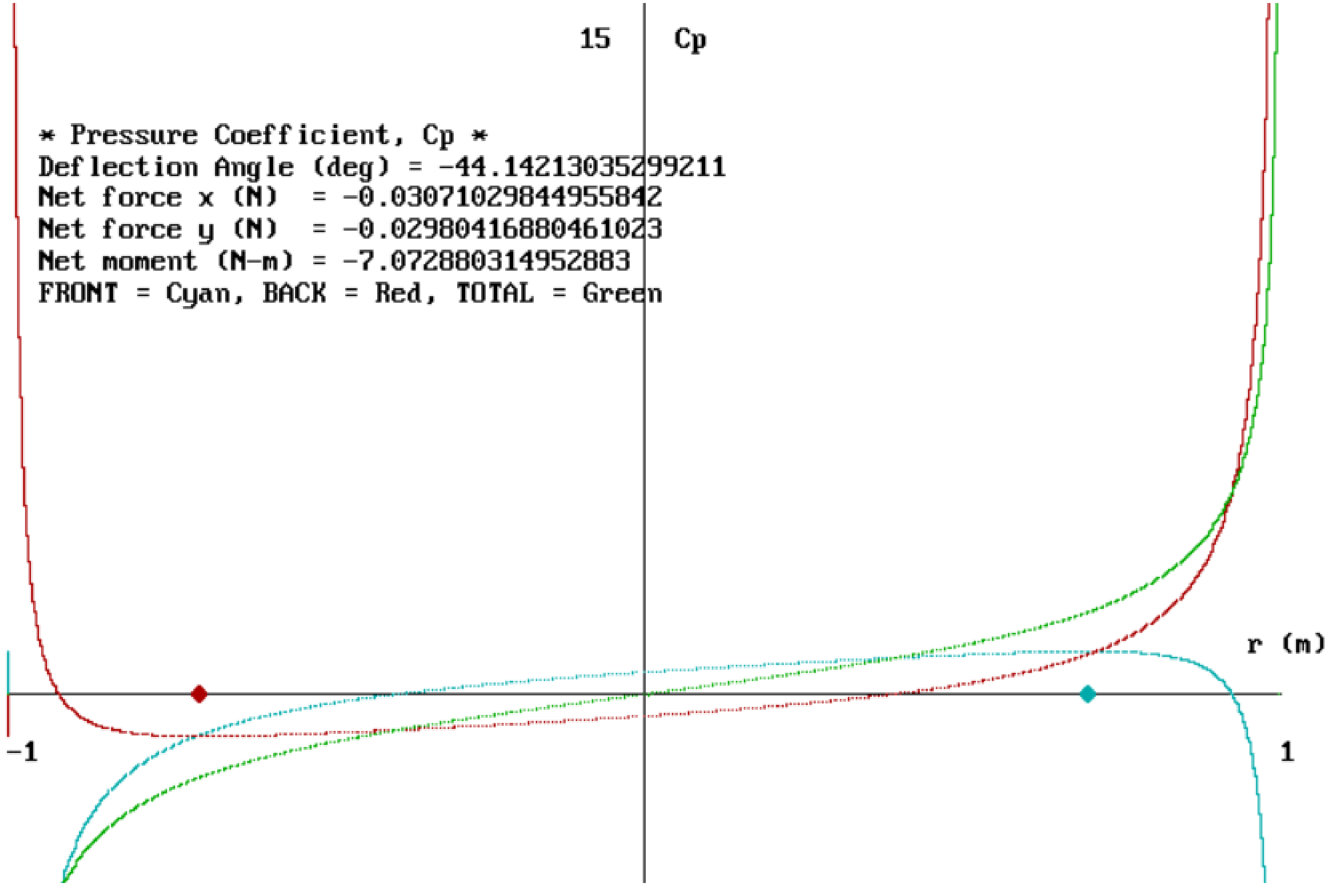
Oscillating Plate, z-6 plane (*left*) and pressure distribution (*right*)





Discussion: Inviscid Flow

- Net force = 0
- Highest force at stagnation points (Cp = 1)
- Negative force at regions of high velocity



Results: Discrete Vortices

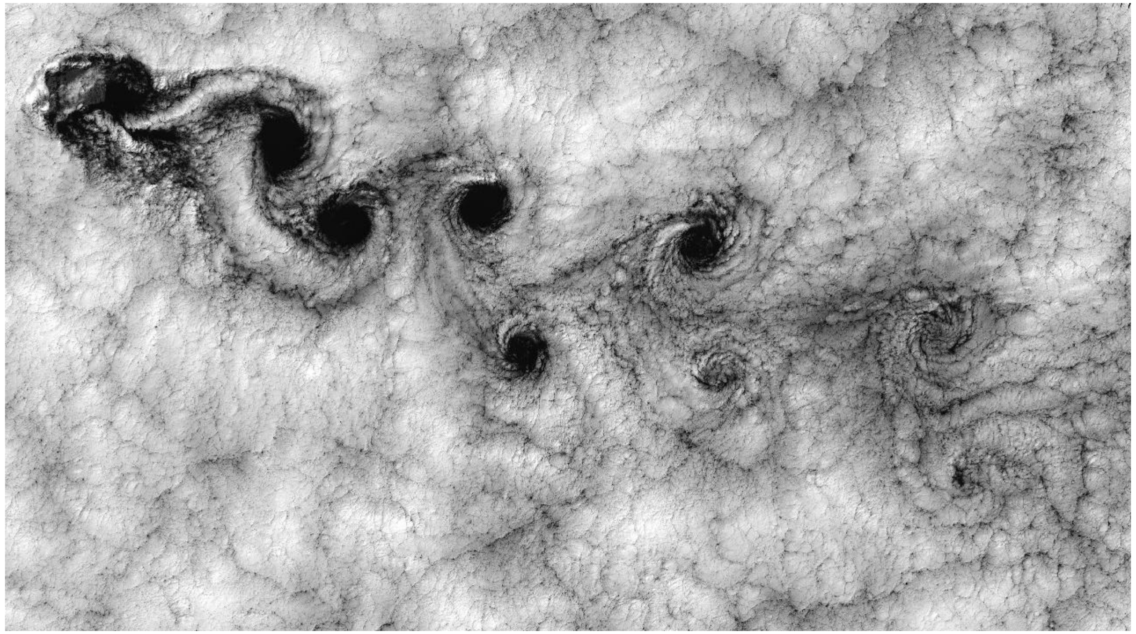
Stationary Plate at 0^o, z6-plane

Z6-Plane
Deflection Angle: 0 (deg)
Initial Angle: 0 (deg)
Number of Vortices: 20
Time (s): -0.125

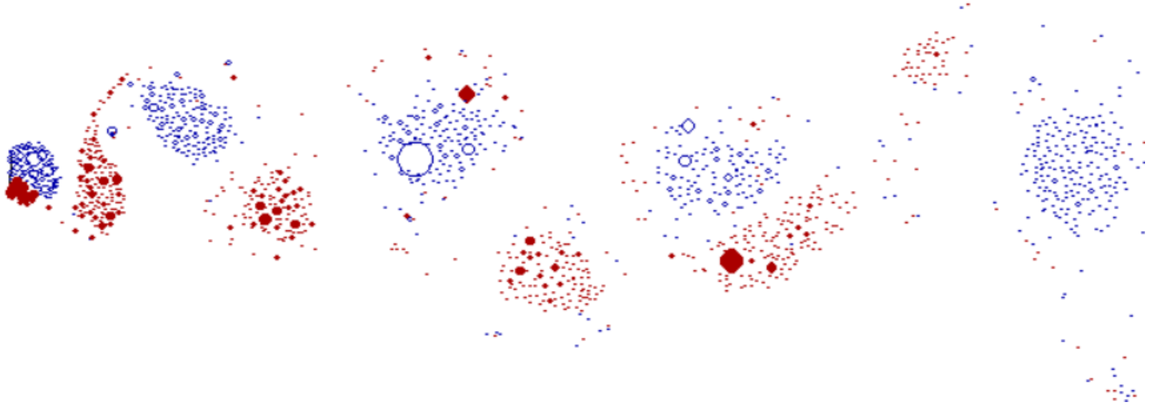
Speed x10

Discussion: Discrete Vortices

- Von Kármán vortex street
- Total circulation = 0 (Kelvin Circulation Theorem)

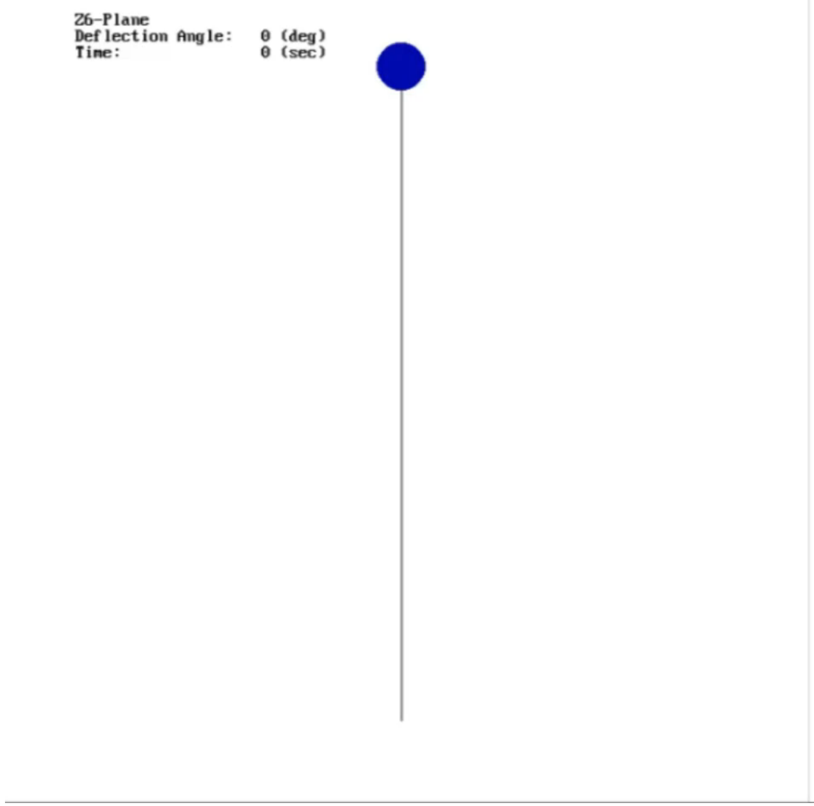


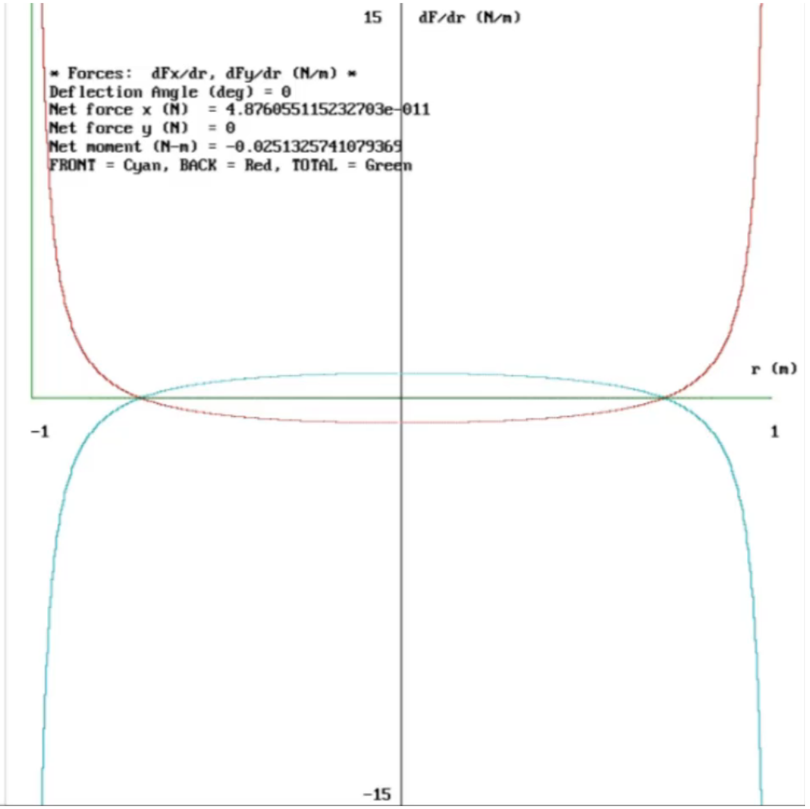
Source: wikimedia.org



Results: Discrete Vortices

Stationary Plate at 0^{o}, z6-plane (*left*) and force distribution (*right*)

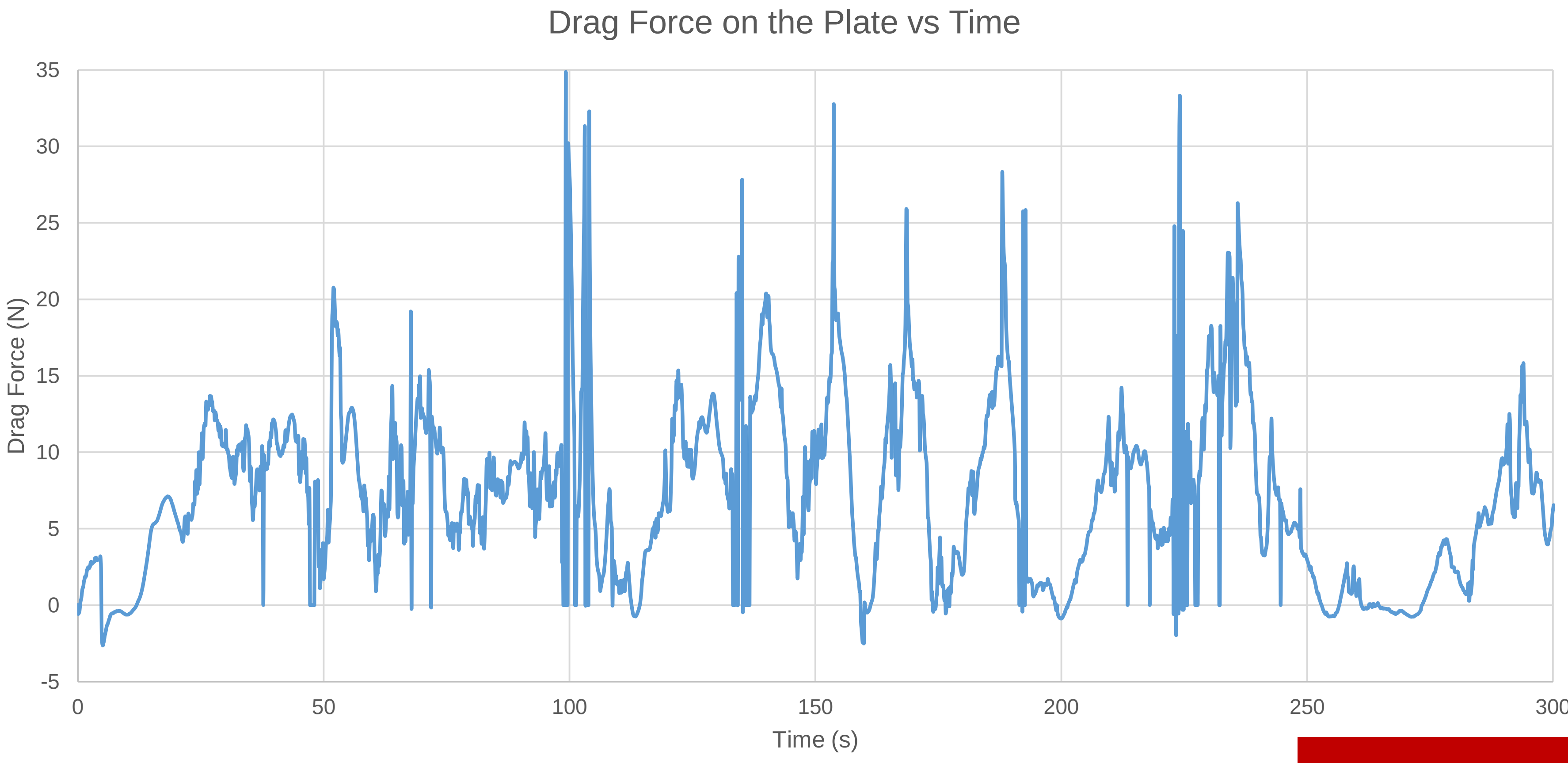




Speed x1

Results: Discrete Vortices

Stationary Plate at 0°



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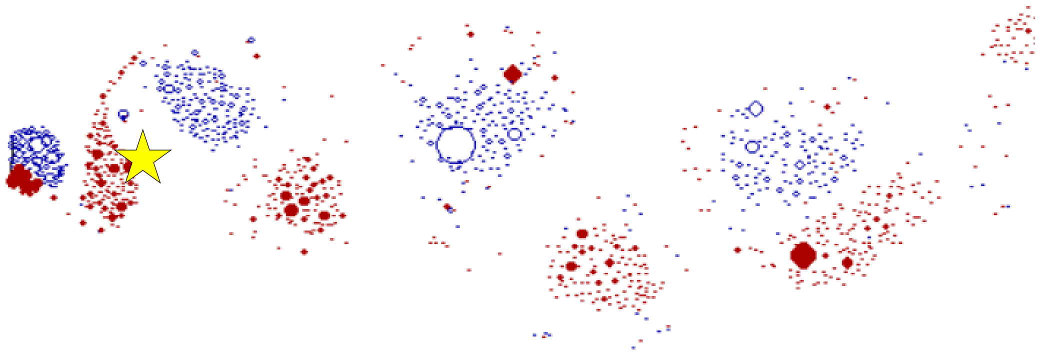
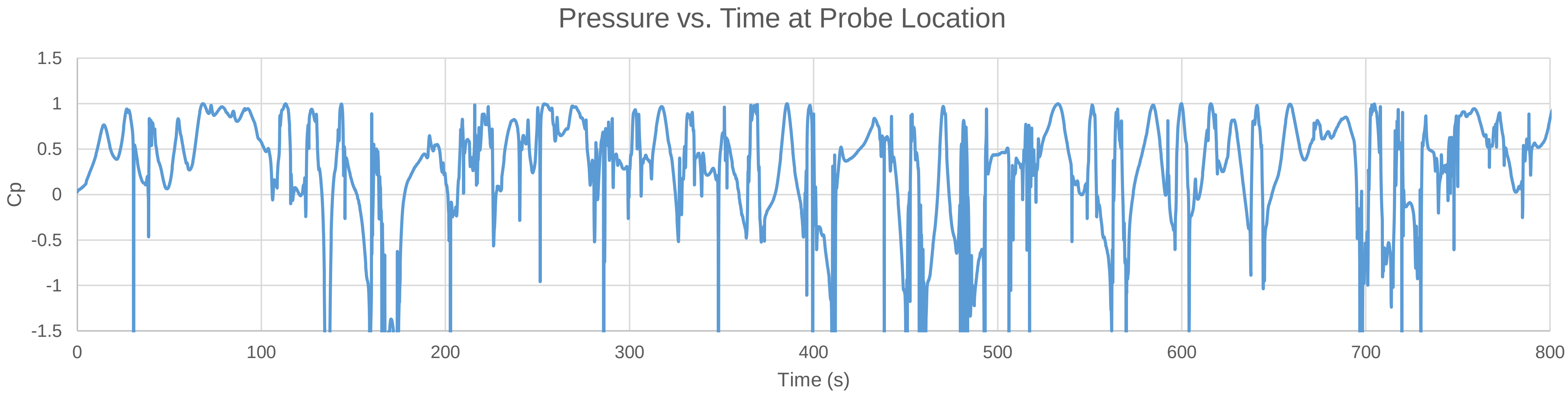
Results: Discrete Vortices



Stationary Plate at 0°



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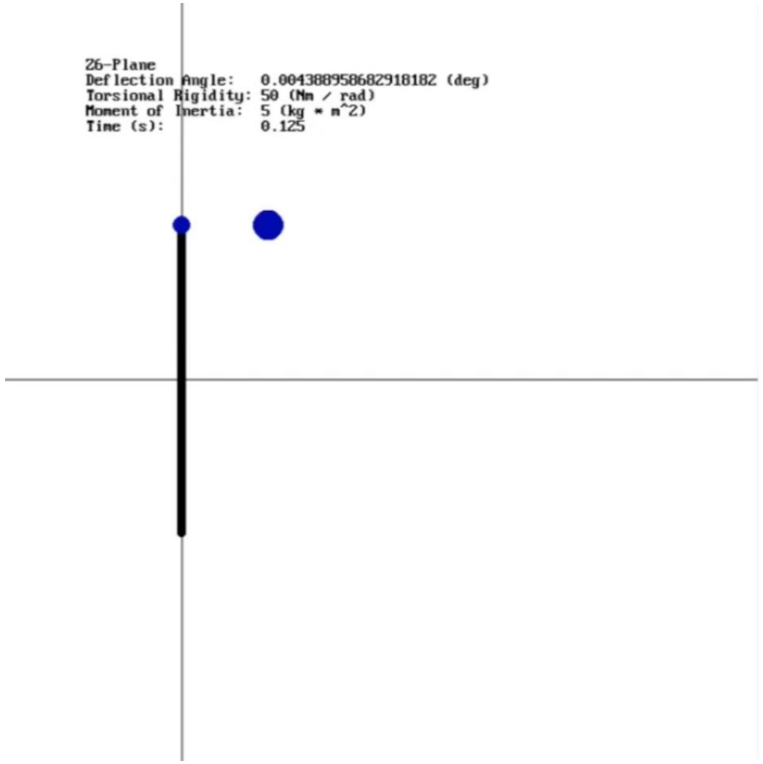
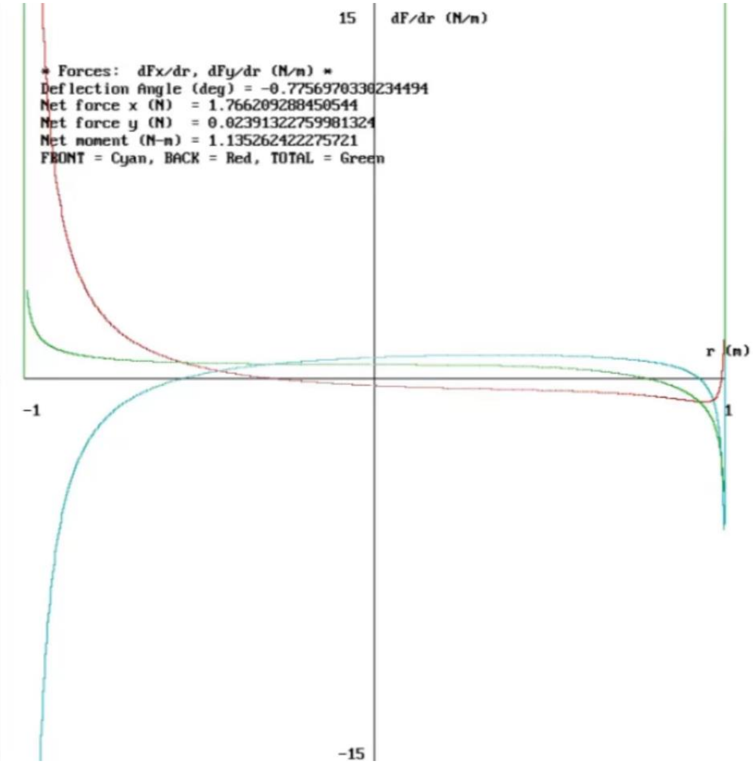
Results: Flutter

Flutter simulation, z6-plane (*left*) and force distribution (*right*)

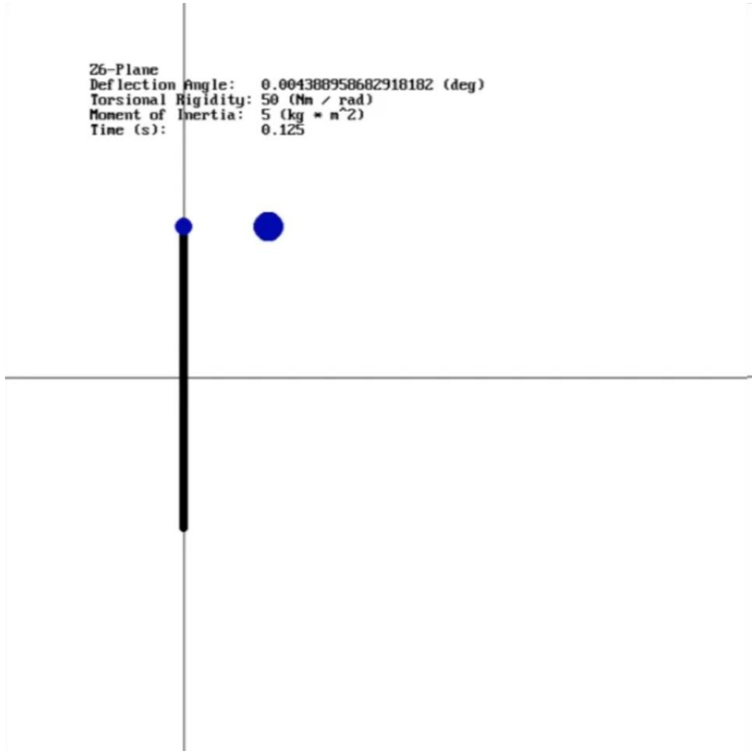


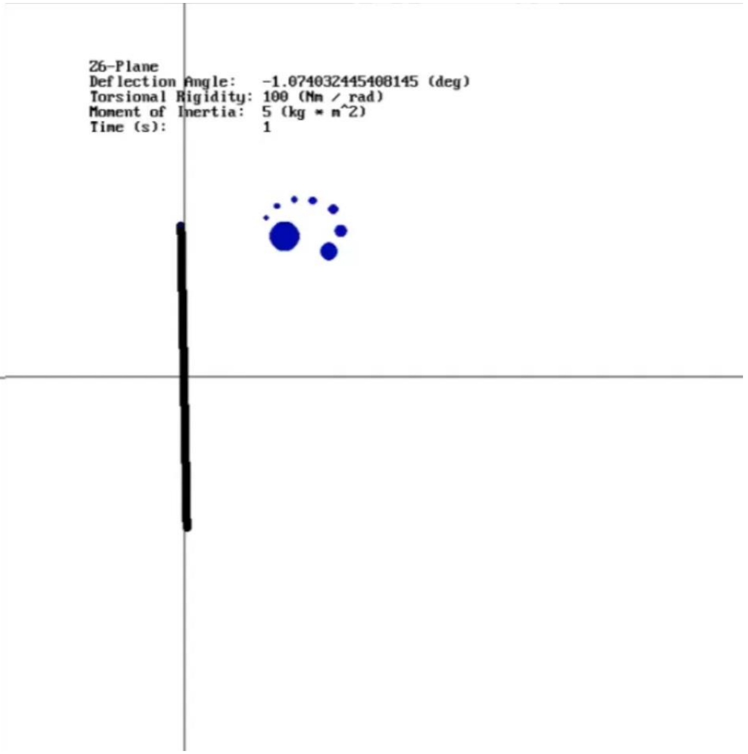


Speed x1

Results: Flutter

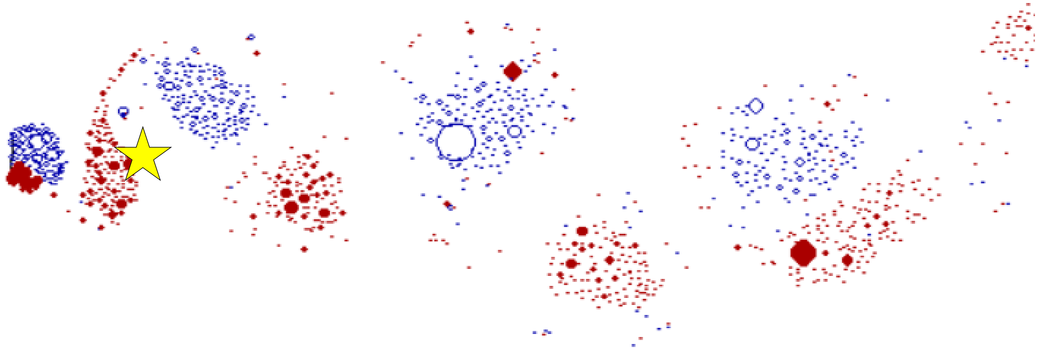
Comparing different torsional stiffness values, less rigid (*left*) and more rigid(*right*)



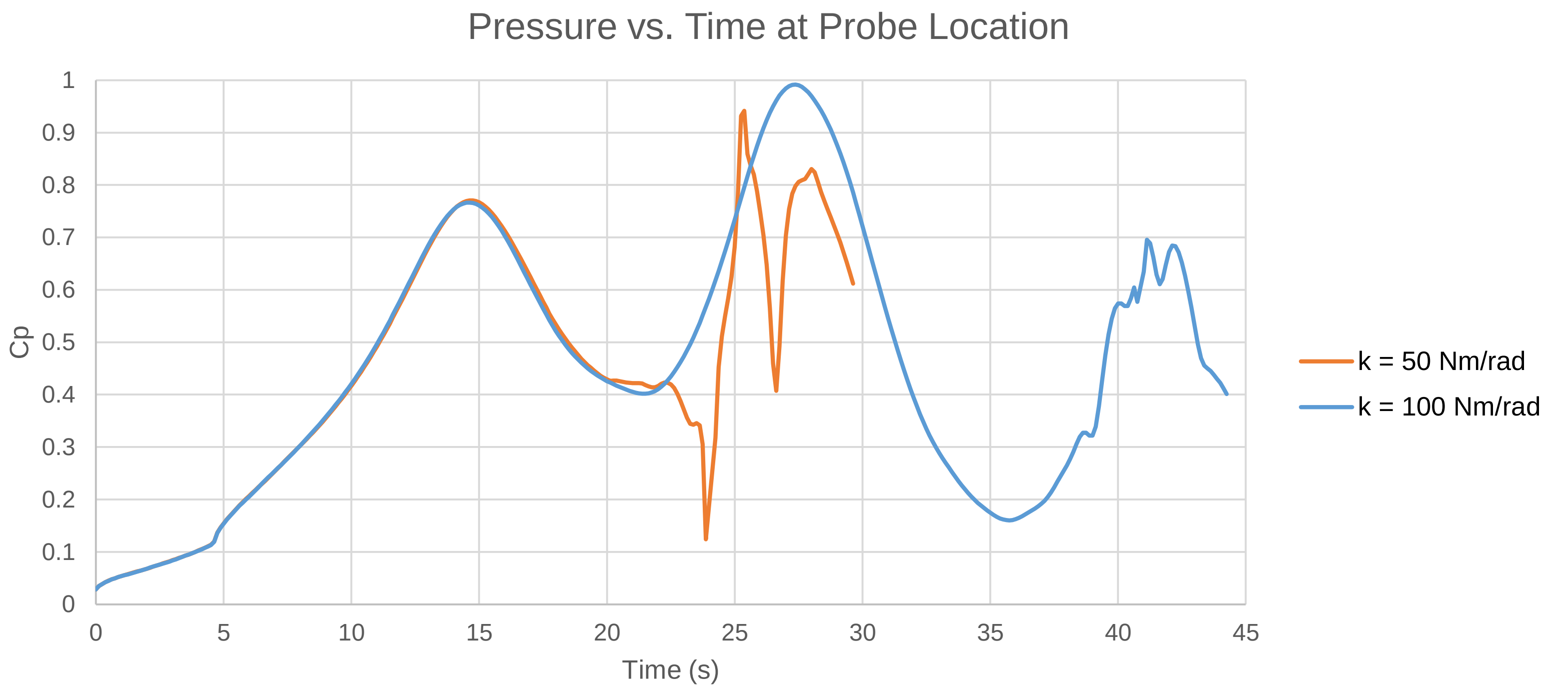


Speed x2

Results: Flutter



Comparing different torsional rigidity values



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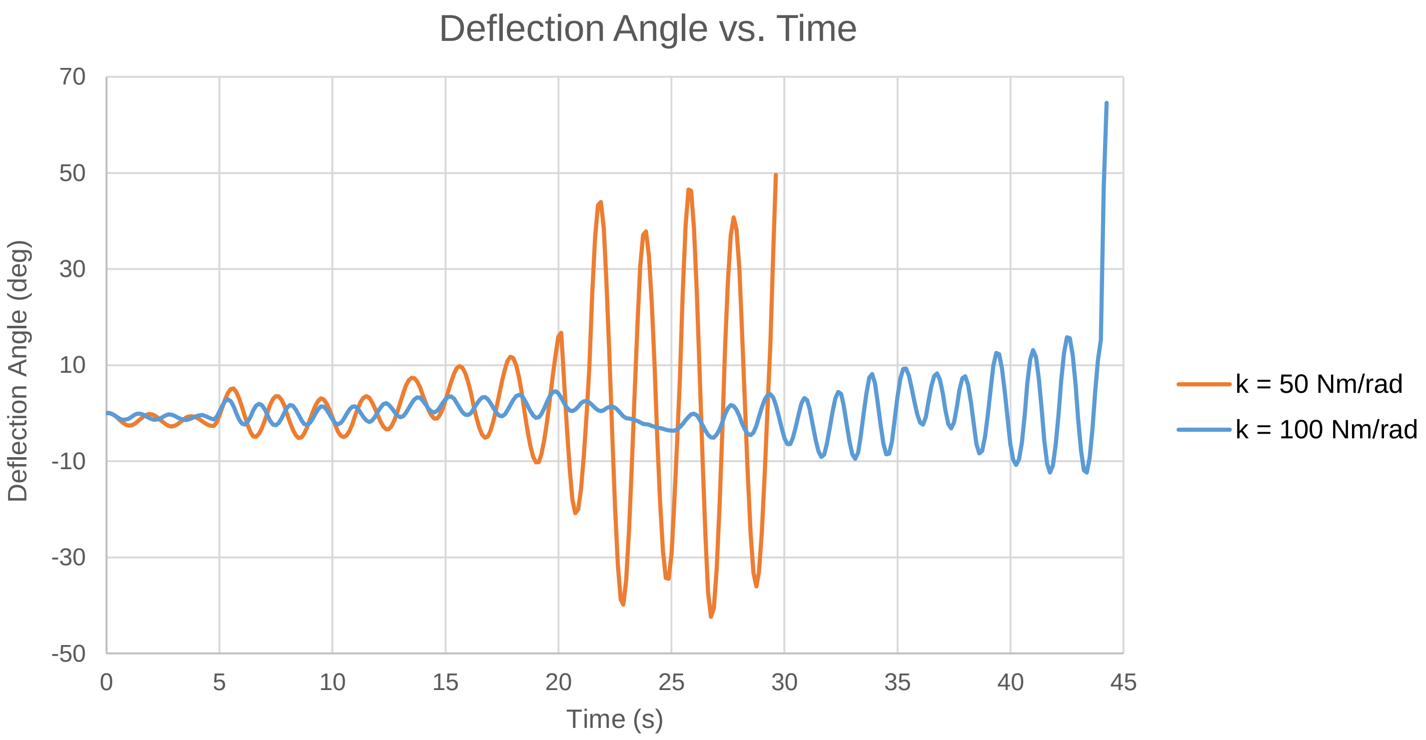
Results: Flutter

Comparing different torsional rigidity values



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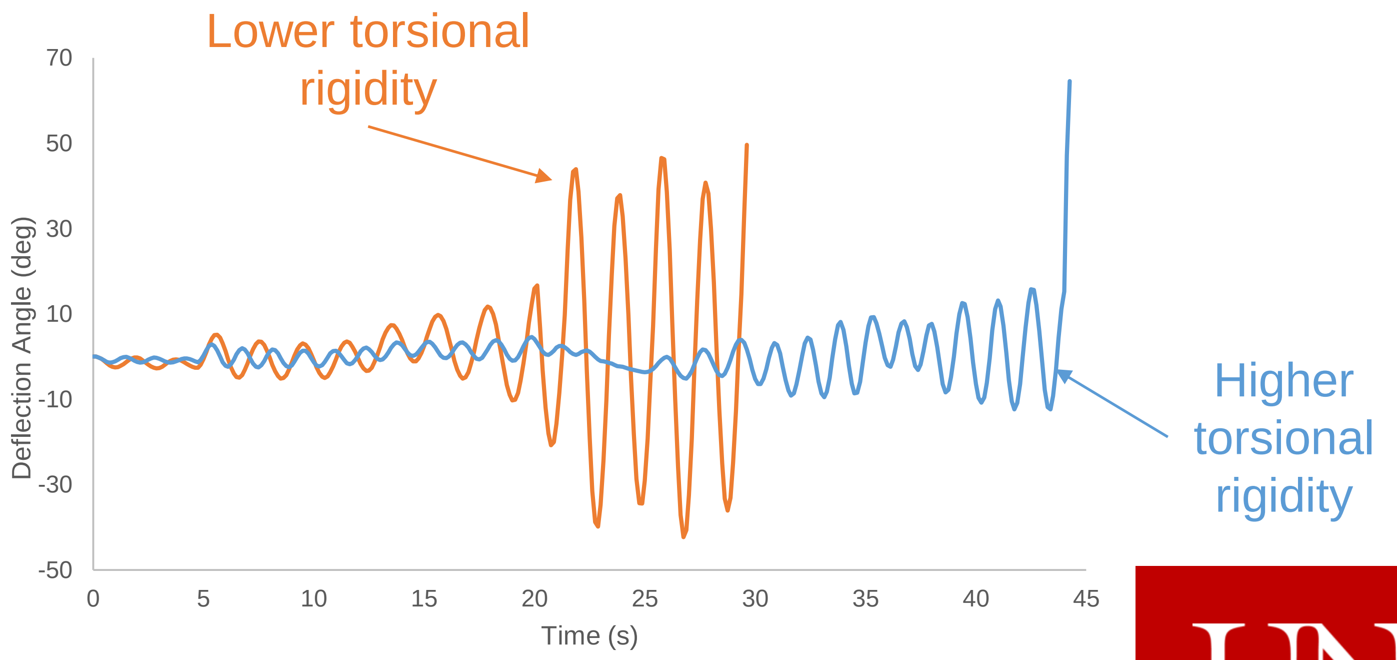
Discussion: Flutter

- Second order differential equation (unstable)
- Higher torsional rigidity corresponds to smaller deflections and longer time until failure



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Conclusion

- DVM provides an efficient method of simulating a simple case of aerodynamic flutter
- The results appear realistic and match the results of previous work and predictions

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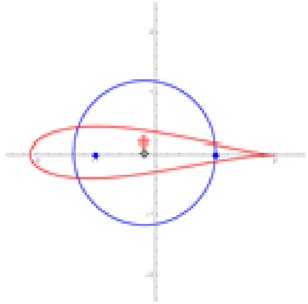
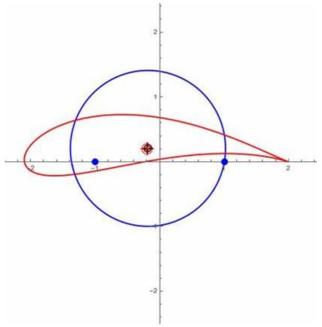
Conclusion: Further Work

- Different approaches to singularities in inviscid flow
  - Free surface theory
  - Sarpkaya's averaging technique
- Examine wake shape
- Fast Fourier Transform on pressure probe data: $St = f(Re)$
- Comparisons with experimental data
- Fatigue considerations
- Conformal mapping to other profiles
  - Streamlined strut
  - Joukowski airfoil
    - Thwaite's Method to find separation points



Source: wolfram.com



Source: wolfram.com

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Questions?

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