QUASI-ONE DIMENSIONAL FLOW THROUGH A NOZZLE WITH A SHOCK

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OVERVIEW

University of California, Irvine

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Masters Thesis

Mechanical and Aerospace Engineering, Fluid Mechanics

Wanted something related to rocket engines

INTRODUCTION

- Van der Waal's Equation of State (EoS) 1873

$$P = \frac{RT}{v-b} - \frac{a}{v^2} \qquad 1 = \frac{P}{\rho RT}$$

- Compressibility Factor - Z

$$Z = \frac{P}{\rho RT}$$

- Redlich-Kwong (1949)

$$P = \frac{RT}{v-b} - \frac{a/T^{0.5}}{v(v+b)}$$

- Soave-Redlich-Kwong (1971)

$$P = \frac{RT}{v-b} - \frac{a(T)}{v(v+b)}$$

WHY USE SOAVE-REDLICH-KWONG

Mainly Algebraic

Minimal Information Needed

Specific Heat Capacity

Critical Pressure

Critical Temperature

Acentric Factor

Molecular Weight

Addresses temperature and pressure variation

Addresses difference of species

PREVIOUS WORKS

- Johannes Diderik van der Waals (1873)
  - Equation everyone knows
- Redlich-Kwong (1949)
  - Shell Development Company
  - Identified new way of approaching temperature dependency
- Soave (1971)
  - Modified Redlich-Kwong
  - Equations used in this study
- Peng Robinson (1976)
  - Another EoS, often compared to Soave-Redlich-Kwong
- Various Works
  - Improvement of binary interaction coefficients
- Huron and Vidal (1979)
  - Defined new mixing rules for gases with multiple species
- Sirignano (2018)
  - Approximate Solution

APPLICATIONS

- Originally developed by Shell company for fracking
- Vapor Equilibrium Studies
- Boilers Design
- Power Cycles
- Wind Tunnel Predictions
- Turbomachinery
- Working Fluids
- Hydraulics
- Shock Waves
- Compressible Flow
- Nozzle Design

SCOPE

Range of properties

Pressure: 10, 30, 50, 100, 200, 500 bar

Temperature: 500, 1000, 2000, 4000 K

Investigating value mores associated with rocket nozzle flow

Some temperature dependent terms not modeled

Eg.

- Specifc heat capacities
- Binary interaction coefficients

ISENTROPIC FLOW

- Need to solve cubic EoS

$$Z = \frac{P}{\rho RT}$$

$$Z^3 - Z^2 + (A - B - B^2)Z - AB = 0$$

- Roots function takes a lot of computational time

- Used Cardano's Method for direct and more efficient calculations

ISENTROPIC FLOW

- Normalized pressure and temperature by stagnation values $P = \frac{T_d}{T_0}$
- Find A and B $T = \frac{T_d}{T_0}$

$$T_{ci} = \frac{T_{ci,d}}{T_0}$$

$$\hat{a} = \frac{aP_0}{(R_u T_0)^2} = \sum_{i=1}^{N}\sum_{j=1}^{N}\sqrt{\hat{a}_i\hat{a}_j}(1-k_{ij})X_iX_j \qquad \hat{a}_i = \frac{aP_d}{(R_uT_d)^2} = 0.42748\frac{T_{ci}^2}{P_{ci}}[1+S_i(1-\sqrt{\frac{T}{T_{ci}}})]^2 \qquad P_{ci} = \frac{P_{ci,d}}{P_0}$$

$$A = \frac{aP_d}{(R_uT_d)^2} = \frac{\hat{a}P}{T^2}$$

$$S_i = 0.48508 + 1.5517\omega_i - 0.15613\omega_i^2$$

$$B = \frac{bP_d}{R_uT_d} = \frac{\hat{b}P}{T}$$

$$\hat{b} = \frac{bP_0}{R_uT_0} = \sum_{i=1}^{N} X_i\hat{b}_i \qquad \hat{b}_i = \frac{b_iP_0}{R_uT_0} = 0.08664\frac{T_{ci}}{P_{ci}}$$

- Now use Cardano's Method
- Solve for Normalized density

$$Z = \frac{P}{T\mathscr{R}} \qquad \mathscr{R} = \frac{\rho R_u T}{P}$$

$$Z^3 - Z^2 + (A - B - B^2)Z - AB = 0$$

ISENTROPIC FLOW

- Choose to vary Pressure

$$\frac{dT}{T} = \beta\frac{d\rho}{\rho} + \frac{1}{c_v + k}ds$$

$$\frac{dP}{P} = (f + g\beta)\frac{d\rho}{\rho} + \frac{1}{c_v + k}ds$$

$$f = \frac{2Z^3 - Z^2 + AB}{Z^3 - B^2Z}$$

$$g = \frac{1}{Z - B} - \frac{A'}{Z(Z + B)}$$

$$\beta = \frac{(\gamma - 1)Zg}{1 + \frac{k}{c_v}}$$

$$\frac{k}{c_v} = (\gamma - 1)\frac{A''}{B}ln\left(\frac{Z + B}{Z}\right)$$

$$\frac{dU^2}{dP} = -\frac{2}{\mathscr{R}}$$

$$C = \sqrt{TZ(f + g\beta)}$$

$$\frac{d\alpha}{dP} = -\alpha[\frac{1}{P(f + g\beta)} - \frac{1}{\mathscr{R}U^2}]$$

ISENTROPIC FLOW

$$A' = \frac{dA}{dT} = T\frac{d\hat{a}}{dT} = \sum_{i=1}^{N}\sum_{j}^{N}\frac{1-k_{ij}}{2}X_iX_jT\left[\sqrt{\frac{\hat{a}_i}{\hat{a}_j}}\frac{d\hat{a}_j}{dT} + \sqrt{\frac{\hat{a}_j}{\hat{a}_i}}\frac{d\hat{a}_i}{dT}\right]$$

$$A'' = \frac{d^2A}{dT^2} = T^2\frac{d^2\hat{a}}{dT^2} =$$

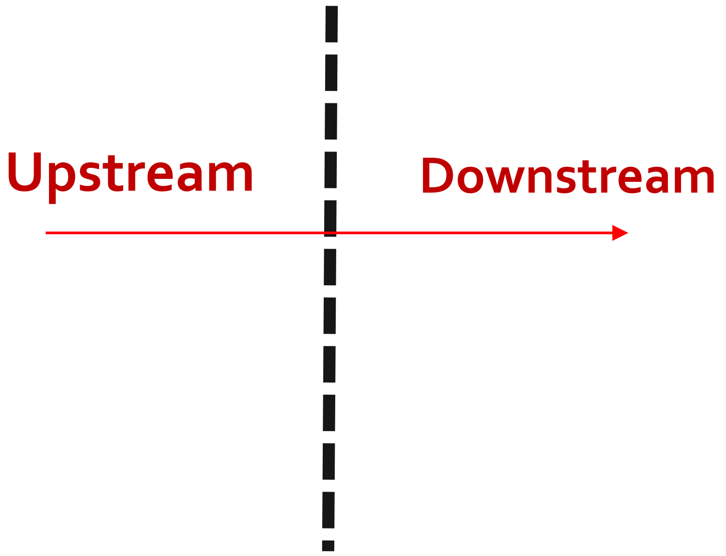
$$\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1-k_{ij}}{2}T^2X_iX_j\left[\sqrt{\frac{\hat{a}_i}{\hat{a}_j}}\frac{d^2\hat{a}_j}{dT^2} + \frac{1}{\hat{a}_i\hat{a}_j}\frac{d\hat{a}_i}{dT}\frac{d\hat{a}_j}{dT} - \frac{1}{2}\frac{\hat{a}_i{}^{0.5}}{\hat{a}_j{}^{1.5}}\frac{d\hat{a}_j{}^2}{dT^2} + \sqrt{\frac{\hat{a}_j}{\hat{a}_i}}\frac{d^2\hat{a}_j}{dT^2} - \frac{1}{2}\frac{\hat{a}_j{}^{0.5}}{\hat{a}_i{}^{1.5}}\frac{d^2\hat{a}_i}{dT^2}\right]$$

$$\frac{d\hat{a}_i}{dT} = 0.42748\frac{T_{ci}^{1.5}}{P_{ci}T^{0.5}}S_i\left[1 + S_i(1 - \sqrt{\frac{T}{T_{ci}}})\right]$$

$$\frac{d^2\hat{a}_i}{dT^2} = -\frac{0.21374}{P_{ci}}\left(\frac{T_{ci}}{T}\right)^{1.5}S_i(1+S_i)$$

NORMAL SHOCK

- Five properties needed for full understanding of normal shock:
  - Pressure
  - Temperature
  - Velocity
  - Density
  - Enthalpy
- Two methods for normal shock calculation:
  - First upstream velocity not needed
  - Second upstream velocity needed



NORMAL SHOCK – METHOD ONE

- Ten total properties
- 4 Known
  - T1, P1, Z1, h1
- 6 Unknown
  - T2, P1, Z2, h2, U1, U2
- 5 Equations

$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

$$h = h_{ideal} + \frac{1}{MW}\left[R_u T(Z-1) + \frac{T\frac{da}{dT} - a}{b} ln \frac{Z+B}{Z}\right]$$

$$h_{ideal} = C_p T$$

$$Z^3 - Z^2 + (A - B - B^2)Z - AB = 0$$

NORMAL SHOCK – METHOD ONE

- Choose Temperature Ratio: T2
- Guess a Pressure Ratio: P2
  - Use Cardano's Method to solve for compressibility: Z2
  - Solve for density: ρ2
  - Manipulate momentum and continuity to solve for velocity: u1
  - Use continuity to get velocity: u2
  - Using literature defined equation solve for enthalpy:h1
  - Check solution using Total Enthalpy
- If total enthalpy's math T2, P2 combination matches

$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

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NORMAL SHOCK – METHOD TWO

This time upstream velocity is kno

Guess Tr:

Ignore upstream velocity for now

Go through Method One

Now have all upstream and downstream conditions

Compare U1 found via Method One to U1 defined by upstream conditions

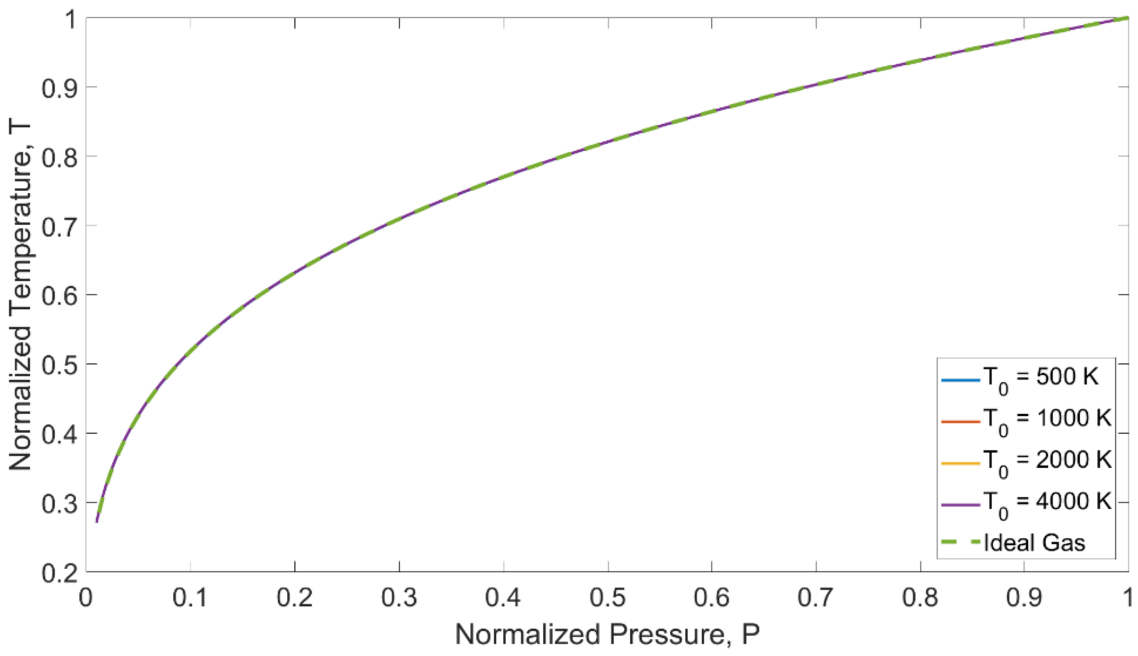
RESULTS COMPUTATIONAL COST

- Use Normalized Values
- Had computer run through all of calculations 52 times

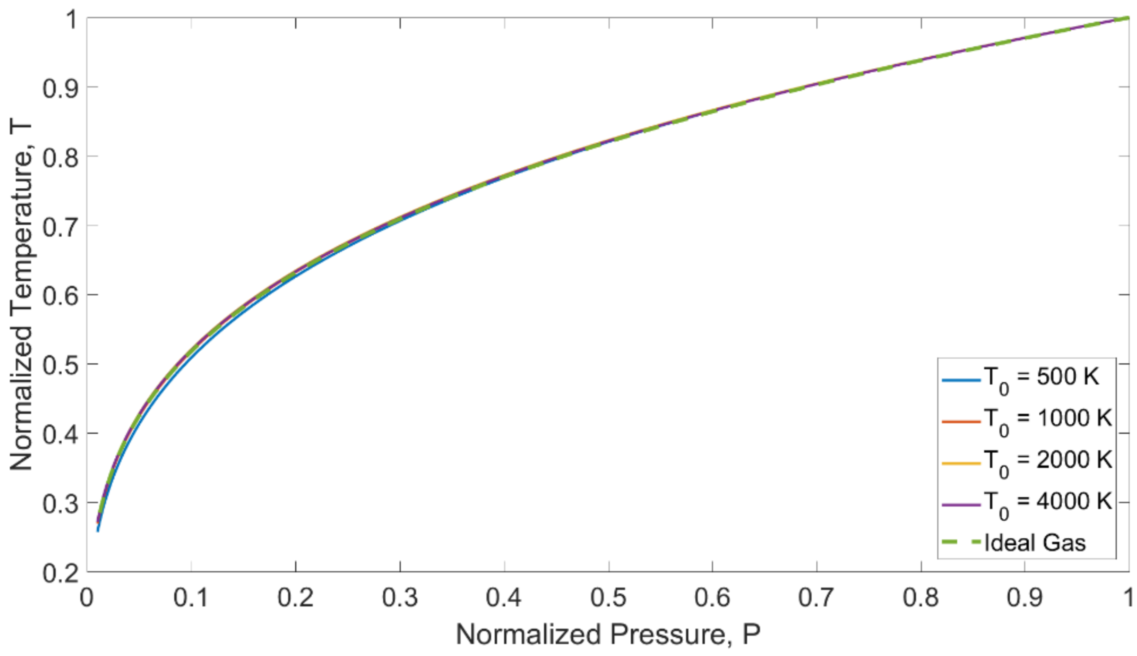
	Root Function	Approximate Solution
Computational Cost	2.0454	0.0327

RESULTS ISENTROPIC FLOW

10 bar



500 bar



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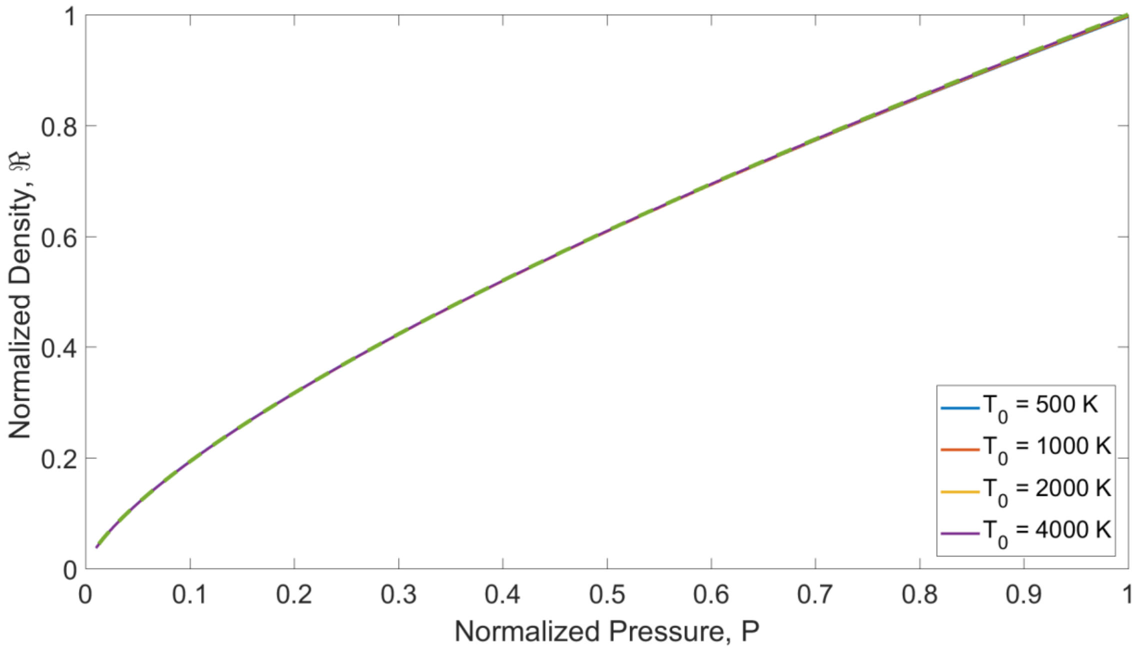

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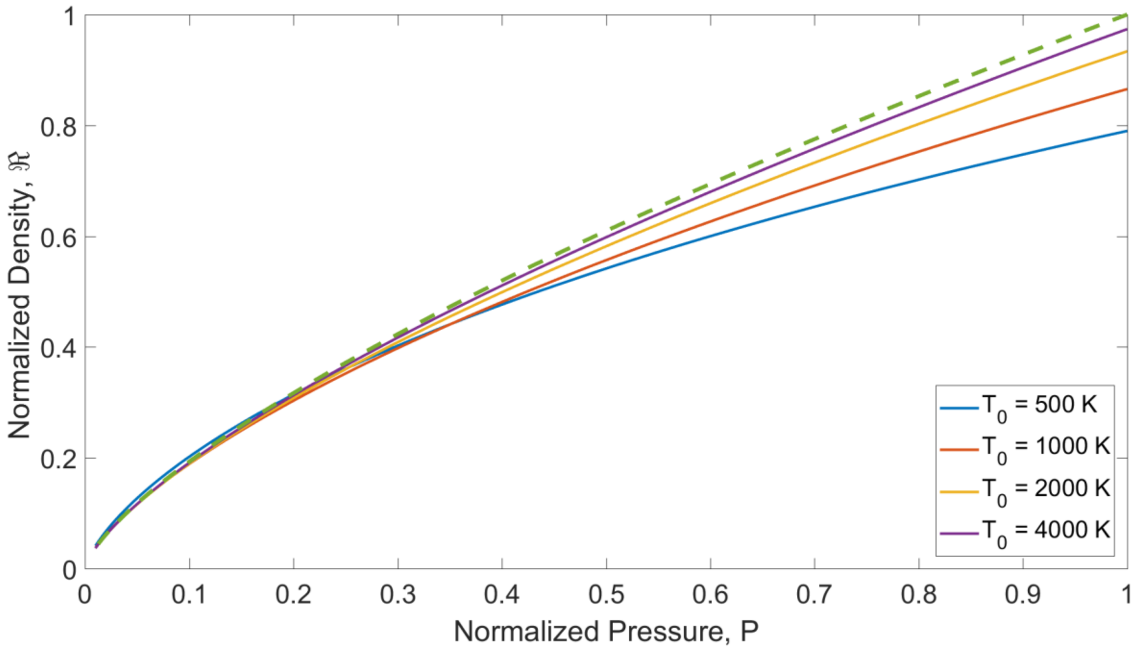

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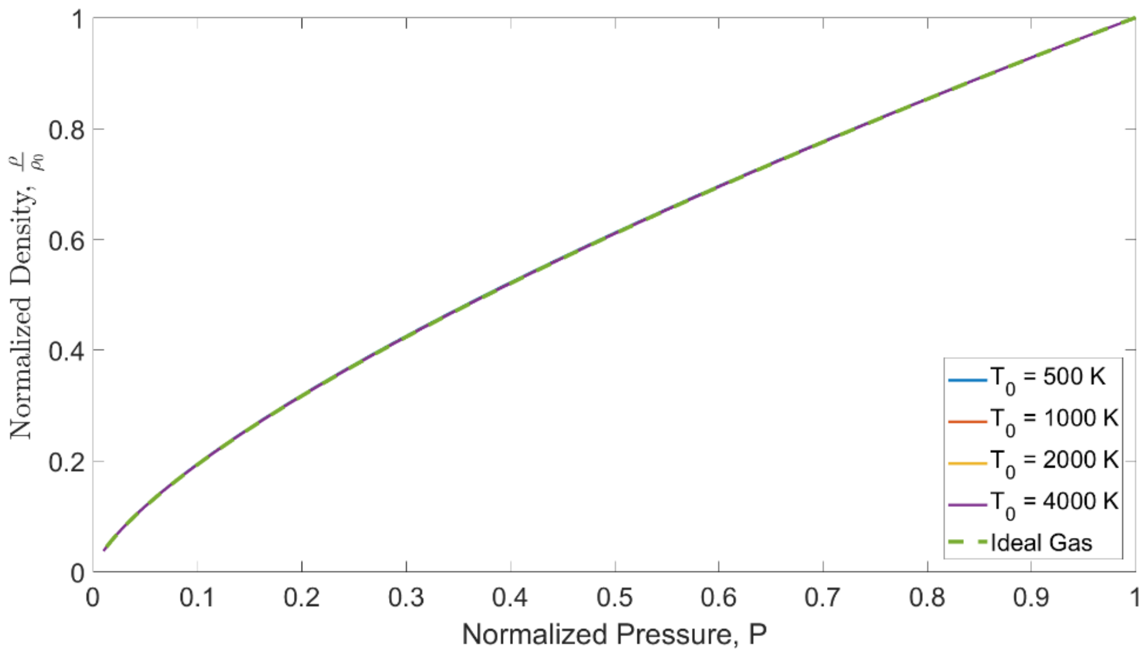

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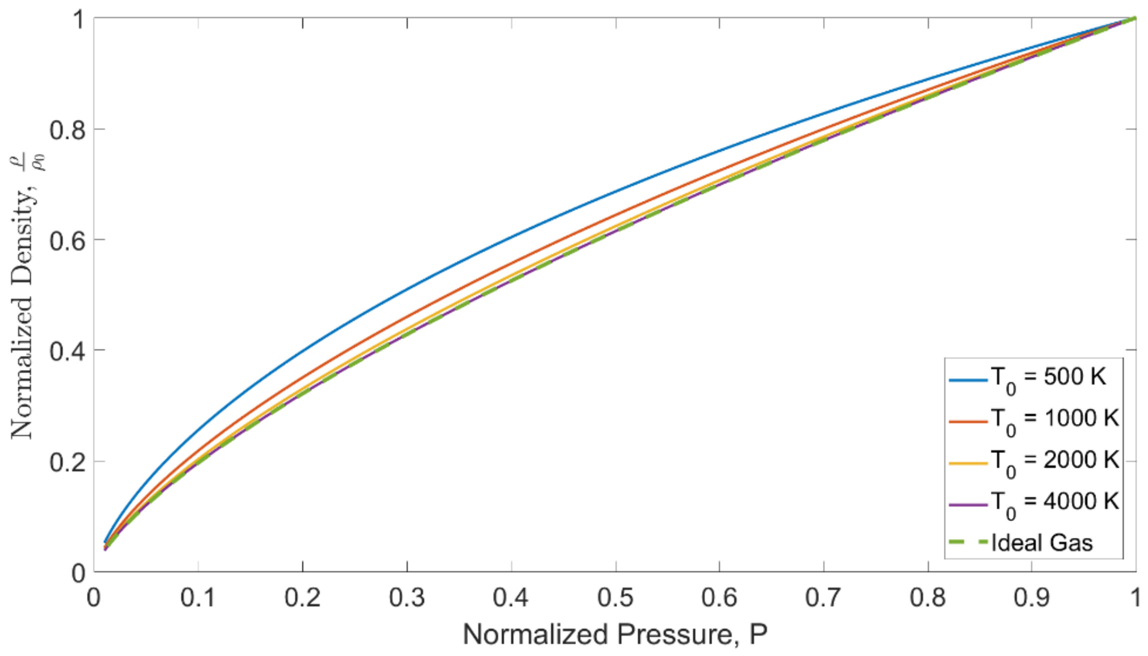

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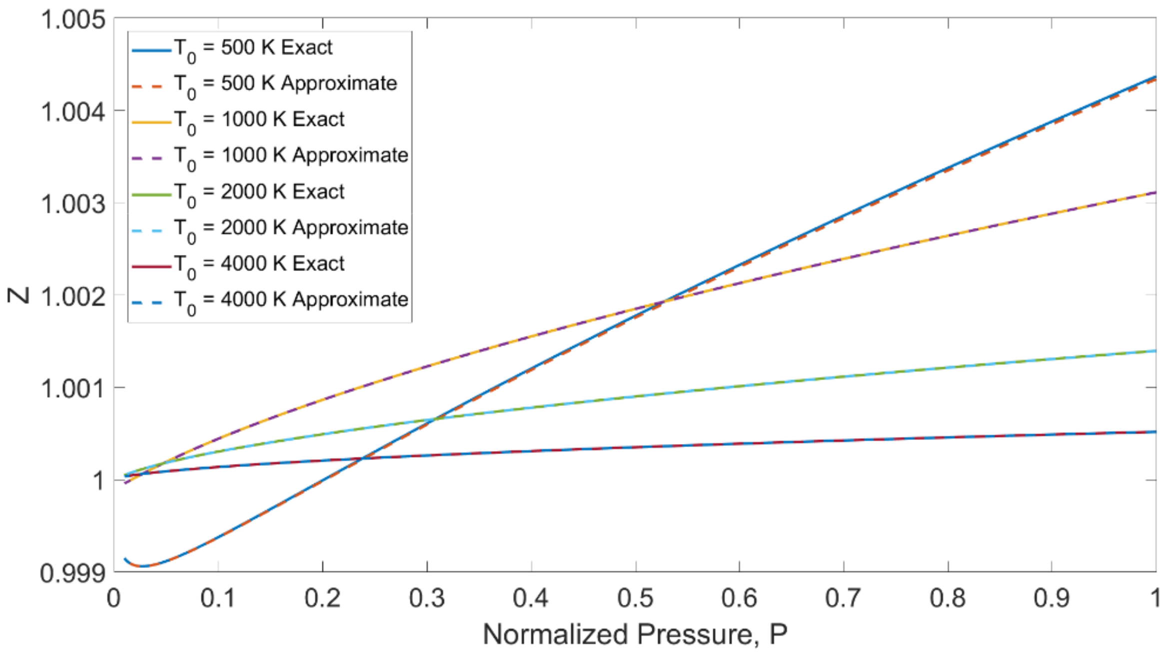

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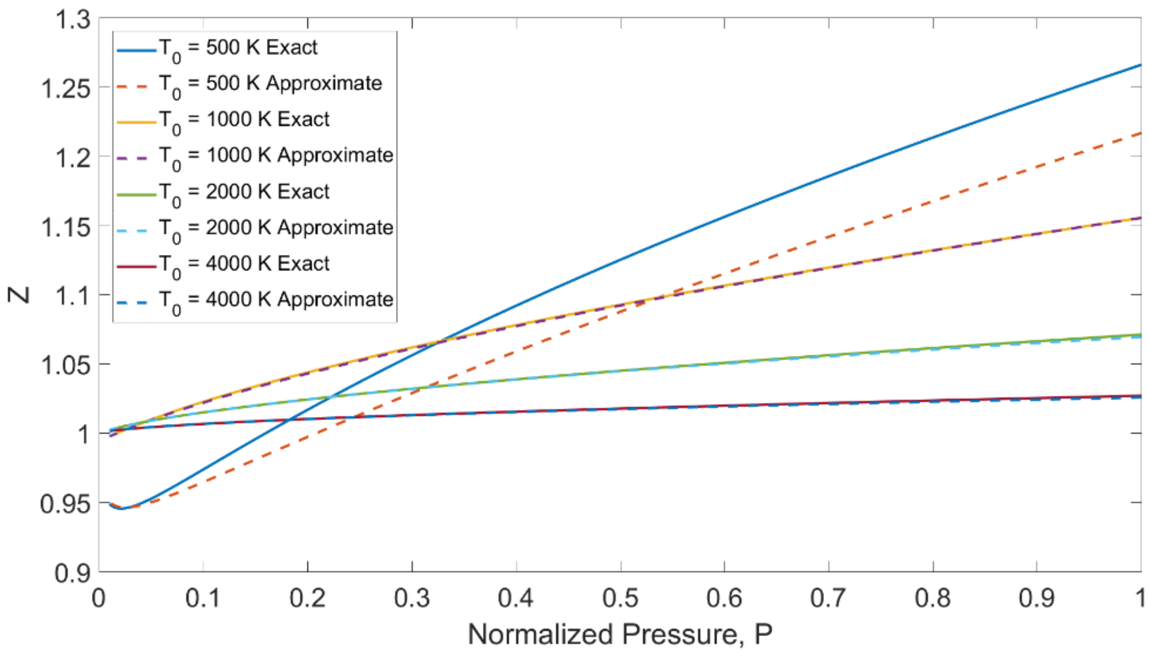

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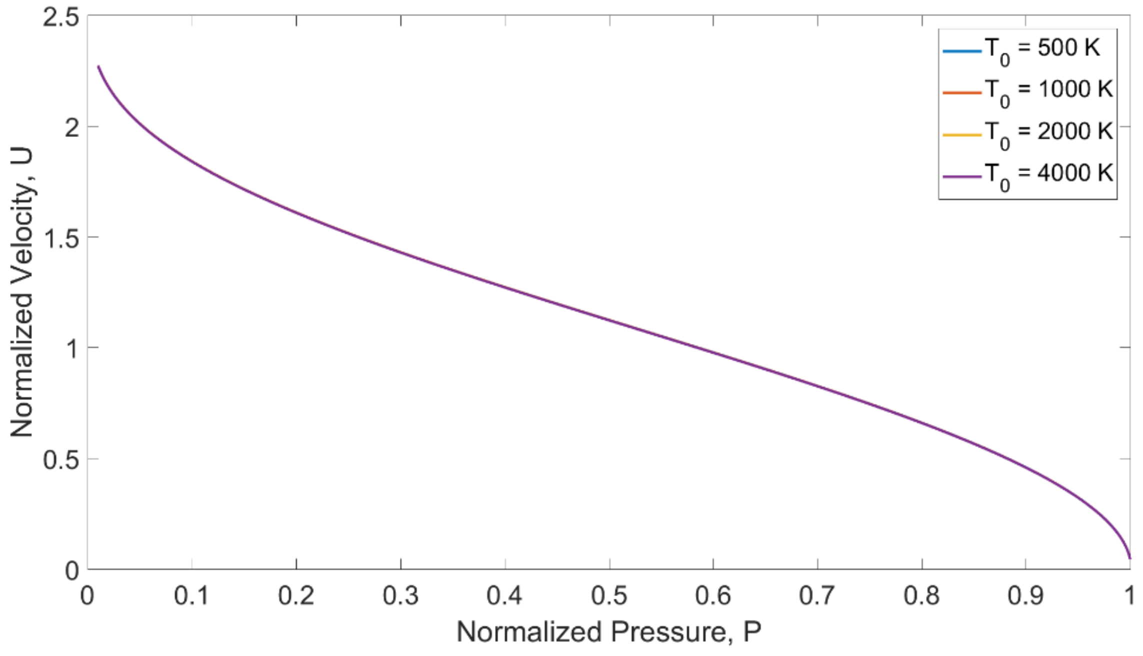

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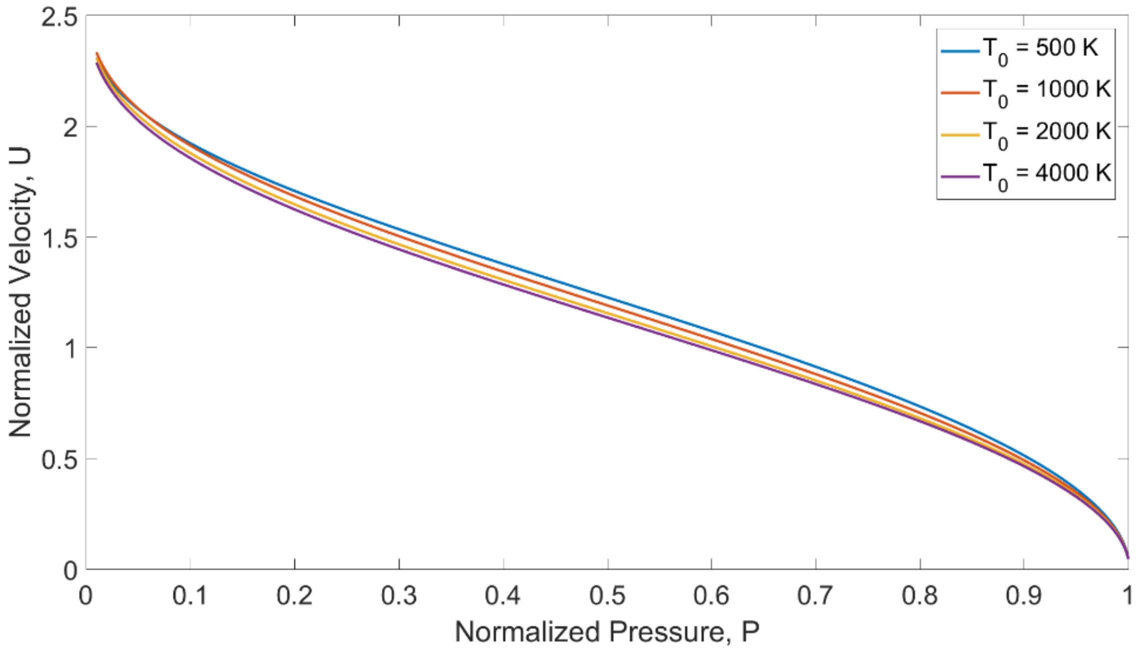

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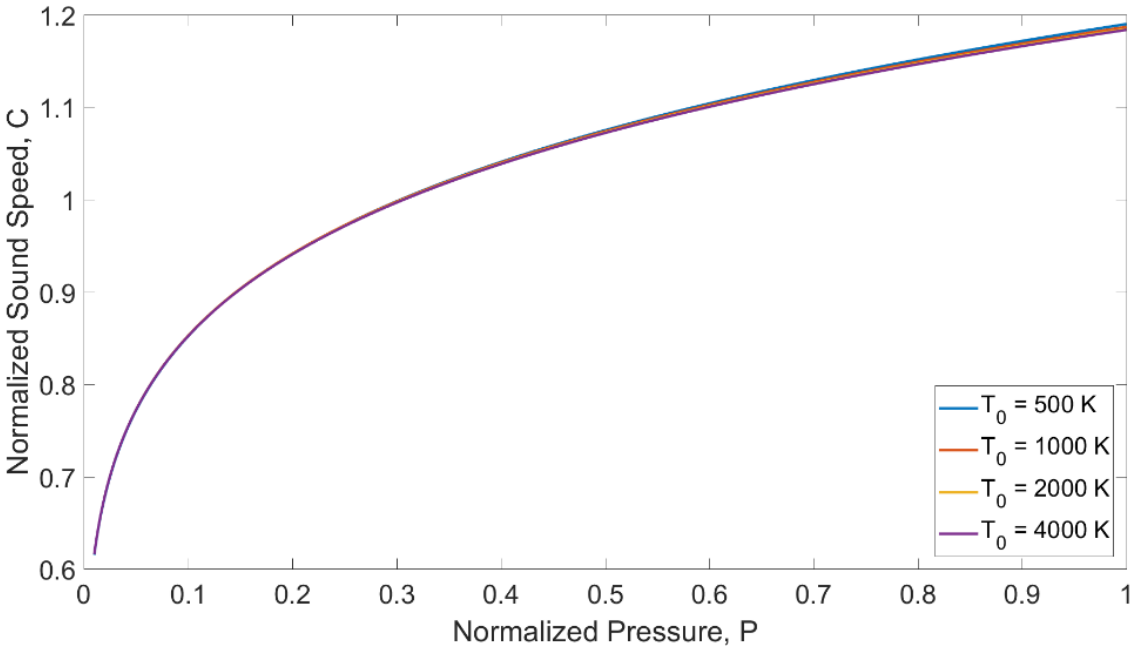

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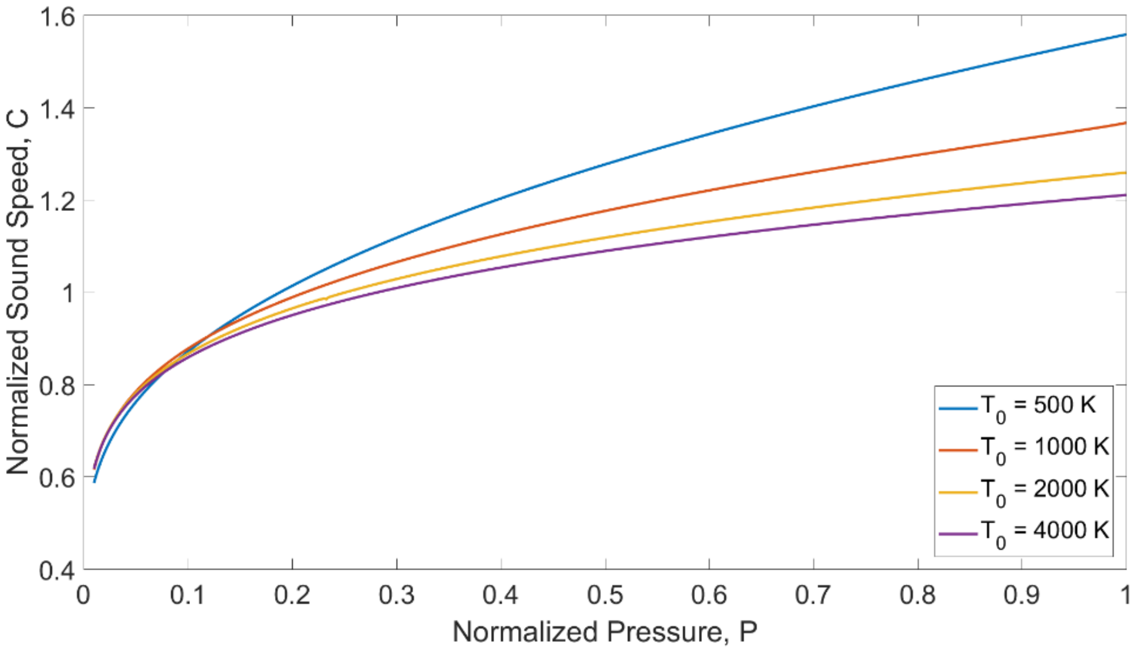

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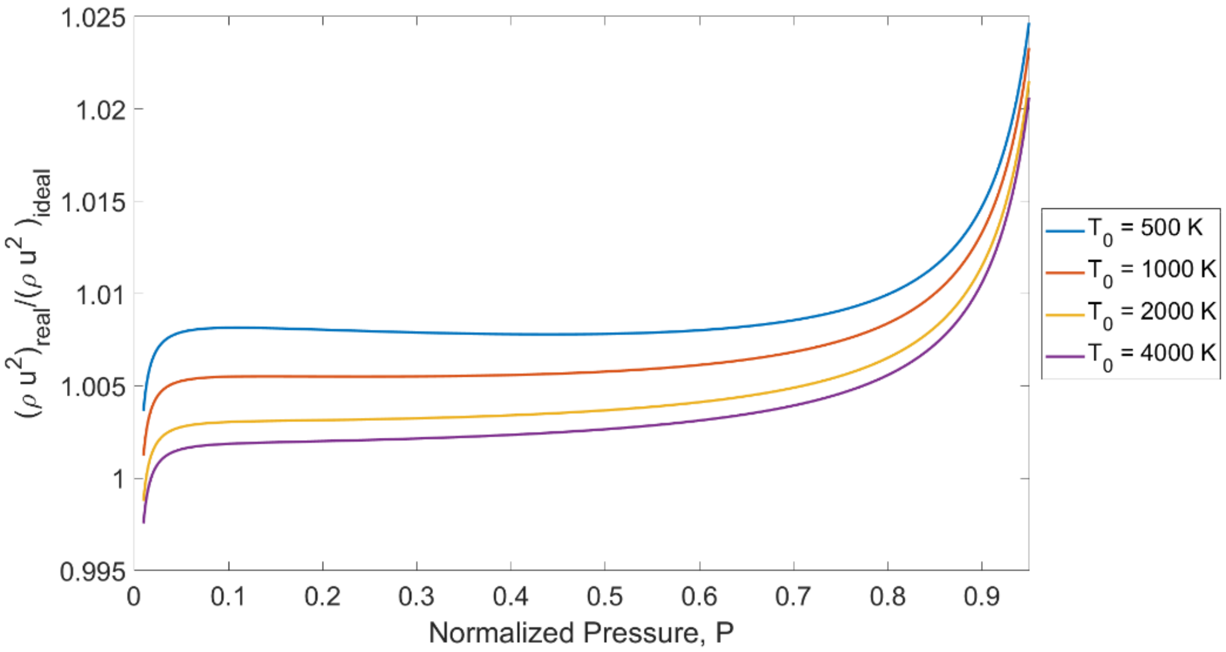

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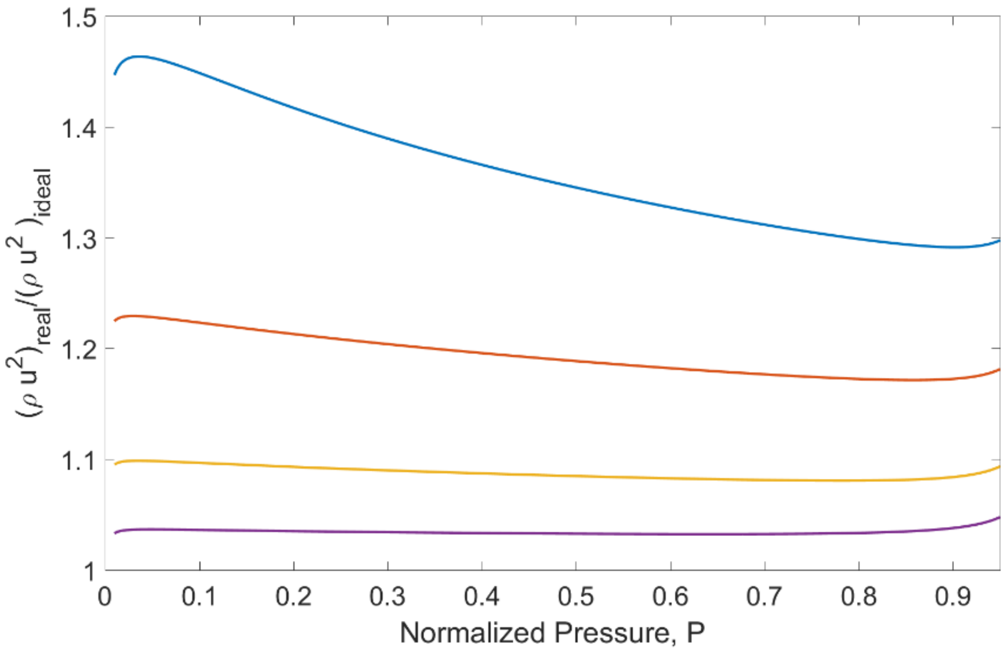
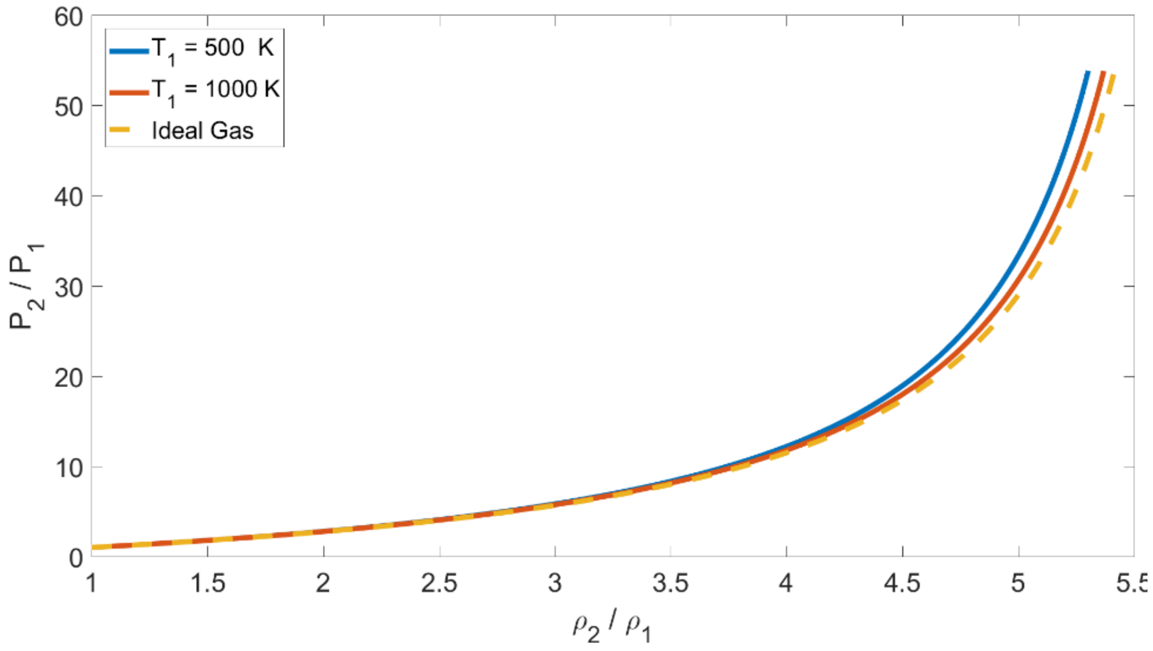


RESULTS NORMAL SHOCK

10 bar



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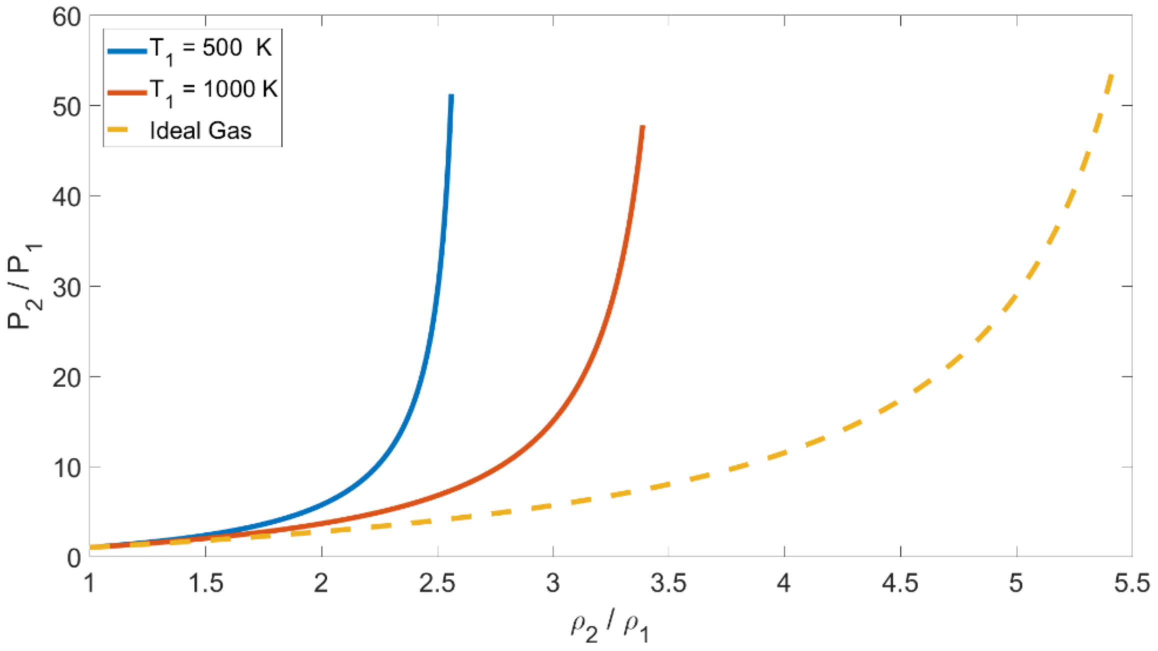

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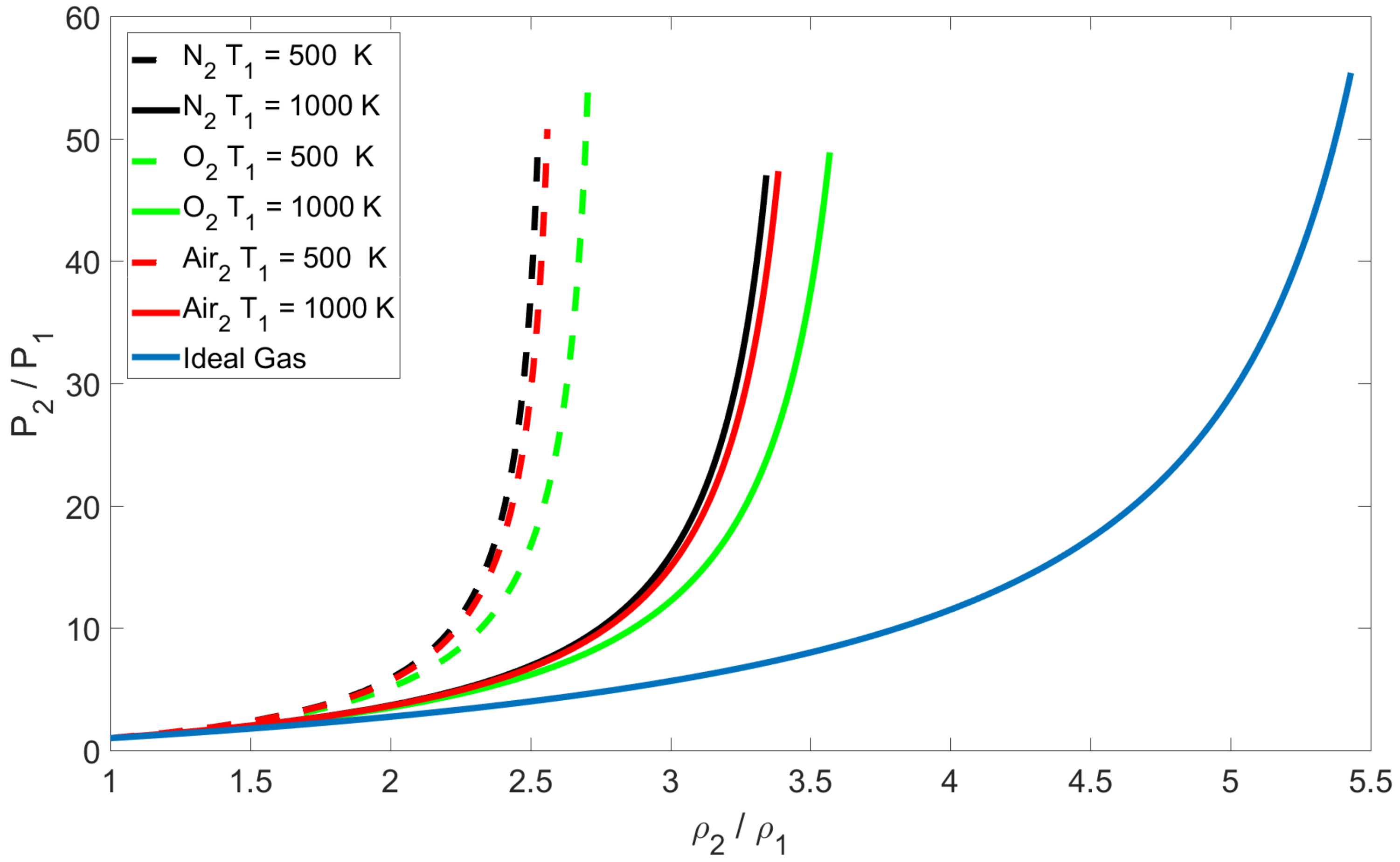

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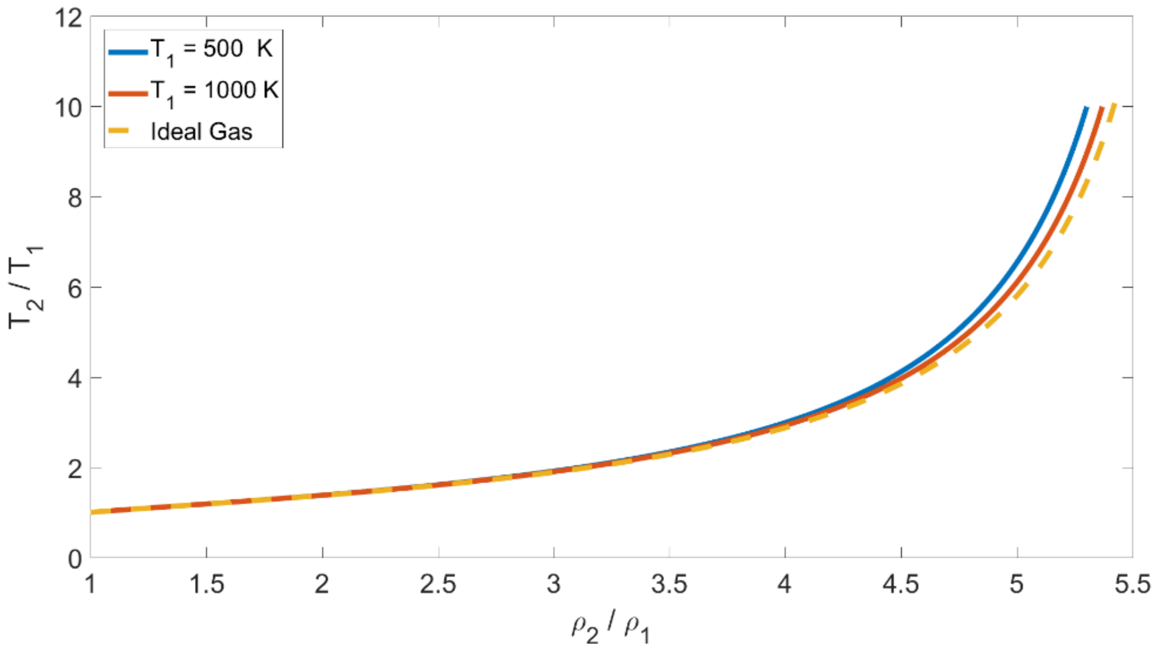

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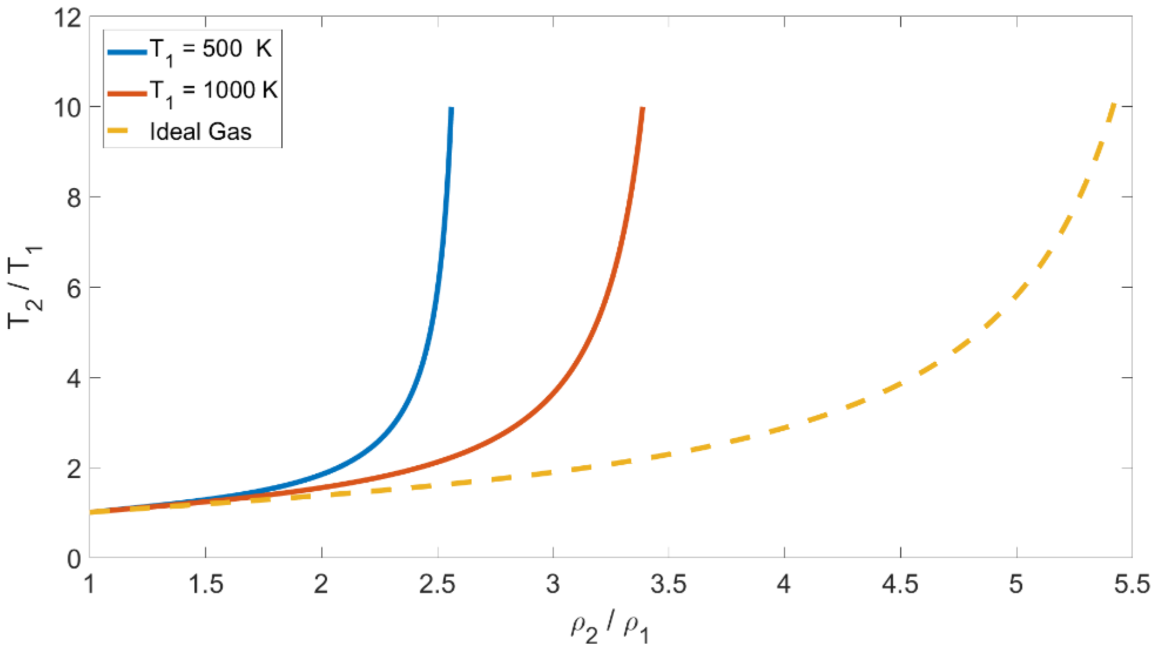

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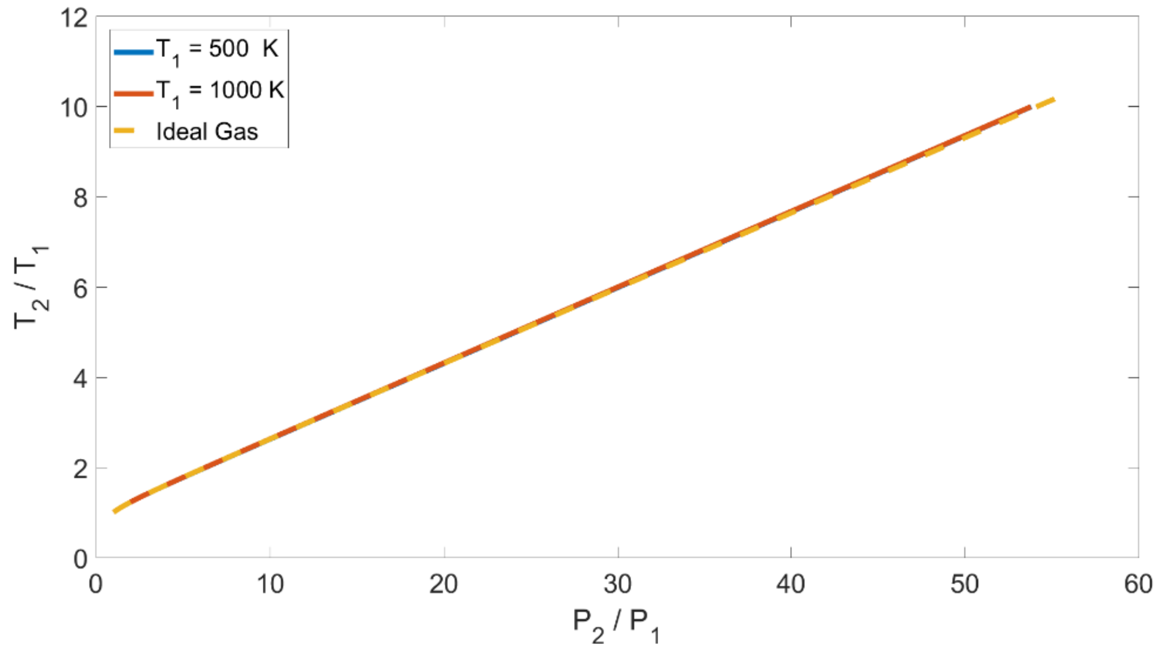

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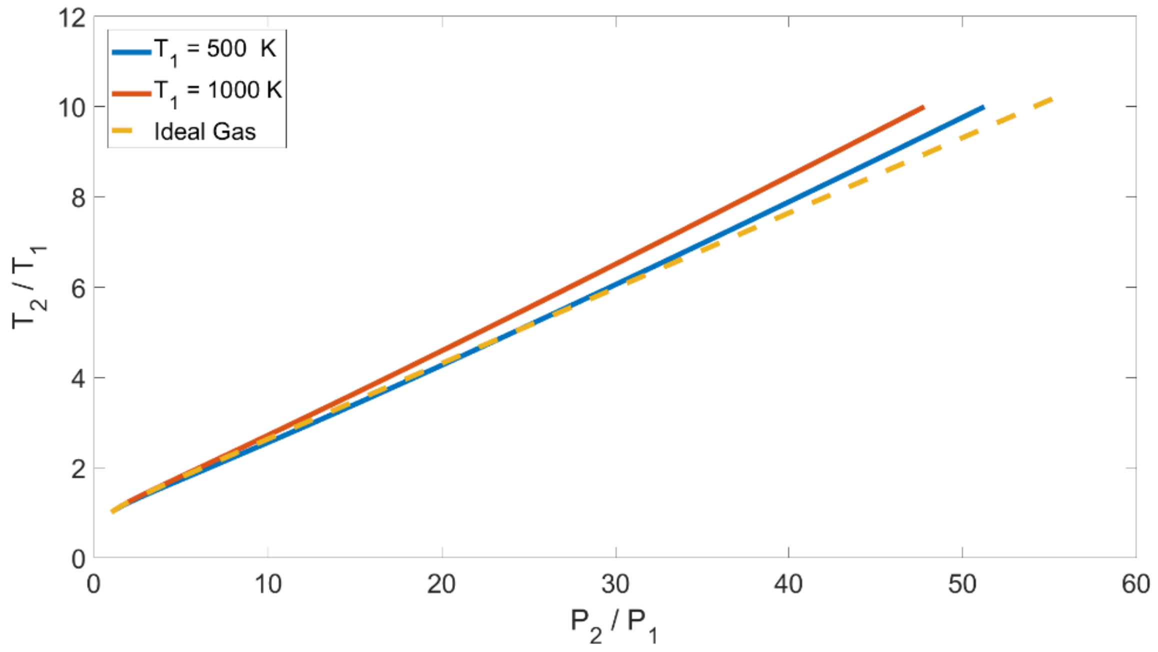

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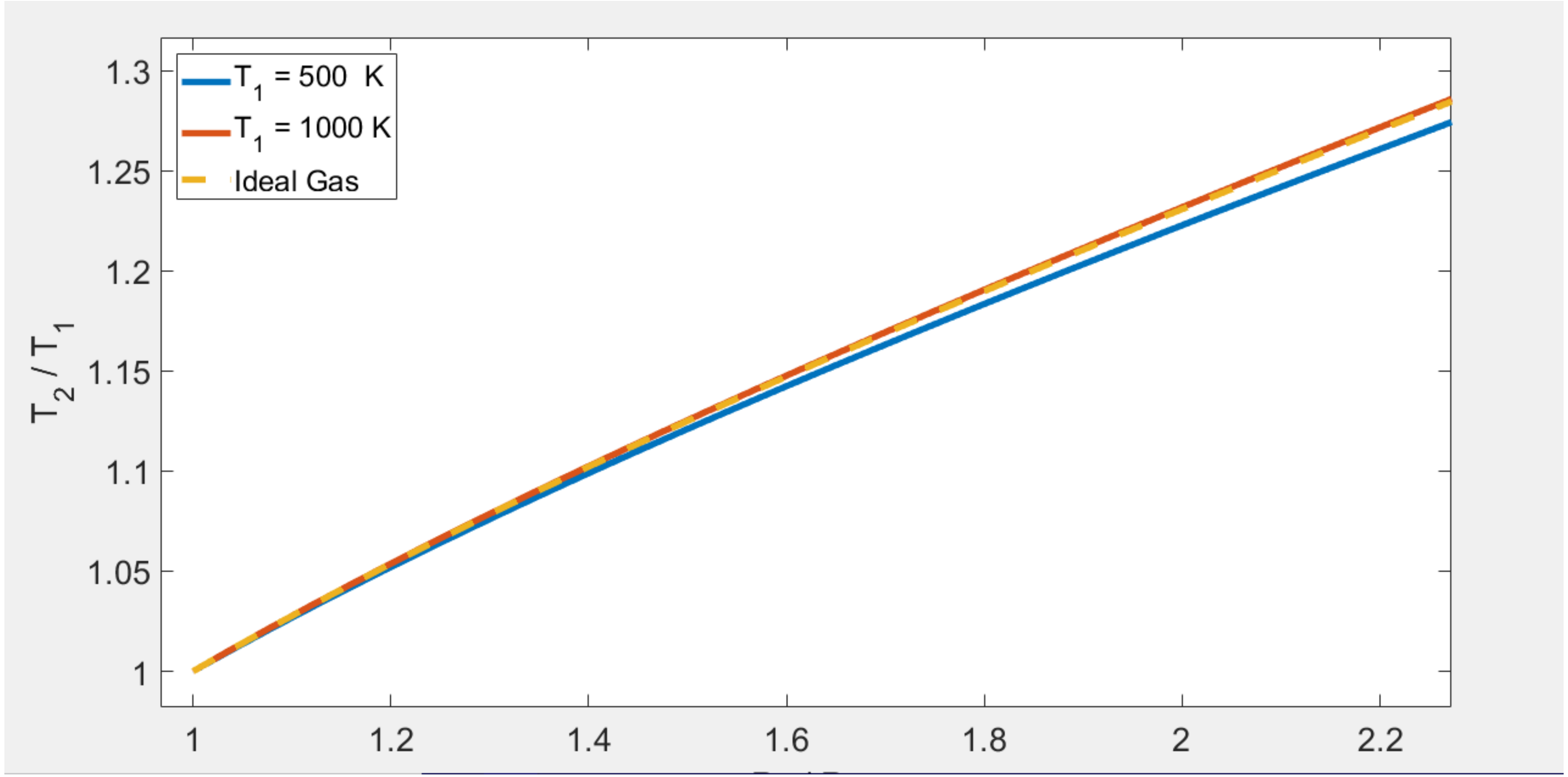

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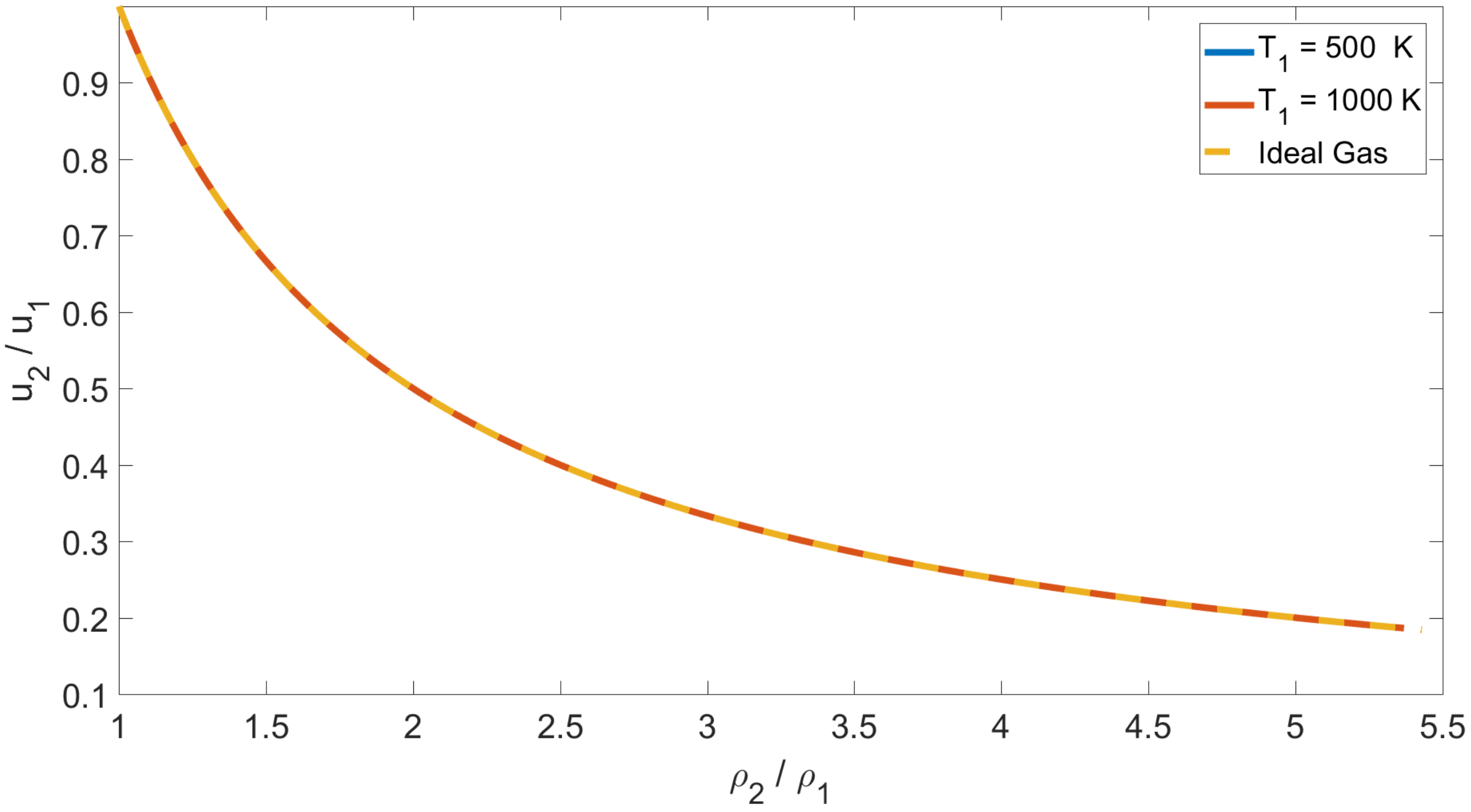

RESULT SUMMARY – MAX DEVIATION

Temperature – 4.25%

Density – 28.19%

Mass Flux – 3.25%

Recap:

Used Soave-Redlich-Kwong EoS

Defined method for solving cubic equation directly

Defined method for calculating isentropic flow

Defined method for finding normal shock

Showed Results

Future Studies:

Focus on higher pressures and temperatures

Explore more values for binary interaction coefficient

Introduce more temperature dependent terms

CLOSING REMARKS

THANK YOU

QUESTIONS