Uncertainty Quantification of Metal Structures

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Uncertainty quantification (UQ) is becoming a more common method for performing engineering analysis, especially in the aerospace industry. A UQ allows for the assessment of reliability of a component to help optimize the design of the structure. While typically more common in such disciplines as thermal analysis or structural analysis of composite materials, examining the uncertainty in a metal structure's analysis can result in large mass savings. A typical structural analysis of a metal structure consists of bounding the aleatory uncertainty of all known analysis variables, while also bounding the epistemic uncertainty by applying the appropriate safety factor (SF) to obtain a minimum margin. This is a conservative approach that has been used historically for the design and analysis of most metal parts. However, assessing the uncertainty of the aleatory variables of an analysis may be useful in certain applications to help reduce the need to account for epistemic uncertainty. This paper will discuss a process that can be used to account for uncertainty in structural analysis, specifically for metal structures. It includes an in-depth discussion of independent variables that can have a great effect on the analysis of such components. This methodology helps to advance the existing processes of analyzing metal components.

I. Introduction

The process for analyzing metal structures has remained relatively unchanged over the past several decades. This process is twofold. First, all aleatory uncertainty, which is the uncertainty due to inherent variation or randomness, is bounded for each analysis variable. This is the standard conservatism seen in most structural analysis, such as applying the worst possible load. The objective of this step is to find the minimum possible margin of safety for a given failure mode. The next step of the process involves accounting for the epistemic uncertainty, or uncertainty due to a lack of knowledge. This uncertainty is typically accounted for by applying the appropriate SF for the given structure. Combining these two steps gives results in a margin for your given structure for a particular failure mode. The margin calculation, shown in equation 1, accounts for the aleatory uncertainty in the "model result" and the epistemic uncertainty in the "SF." While the analysis tools have changed, this overall process has remained constant for analyzing metal structures [1].

$$Margin = \frac{Allowable}{SF(Model\ Result)} - 1 \tag{1}$$

Other disciplines of analysis in aerospace engineering tend to have more uncertainty than metal structures [2]. Metal materials tend to have failure methods that are well understood with plenty of data to support stated material properties. The same cannot be said for such analyses as thermal, and structural analysis of composites and rubbers. Therefore, in order to reduce the uncertainty, and thus create a more accurate prediction of results, a UQ is sometimes performed instead of a typical margin calculation.

While metal structures have relatively less uncertainty compared to the other analyses mentioned previously, performing a UQ on metal components reduces the need to account for epistemic uncertainty. This can result in

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significant mass savings on major structures. Furthermore, a UQ provides a more realistic failure prediction than a typical, conservative analysis.

This paper will discuss a specific process that can be used to account for uncertainty in the analysis of metal structures. It will include an in-depth look into variables that can account for uncertainty in the analysis. The process discussed reduces the inherent uncertainty of such structural analysis, advancing the current methods of how metal parts are analyzed for failure. This process is laid out in Figure 1, and will be discussed in detail. The result of this UQ process is a reliability based SF or predicted maximum deflection.

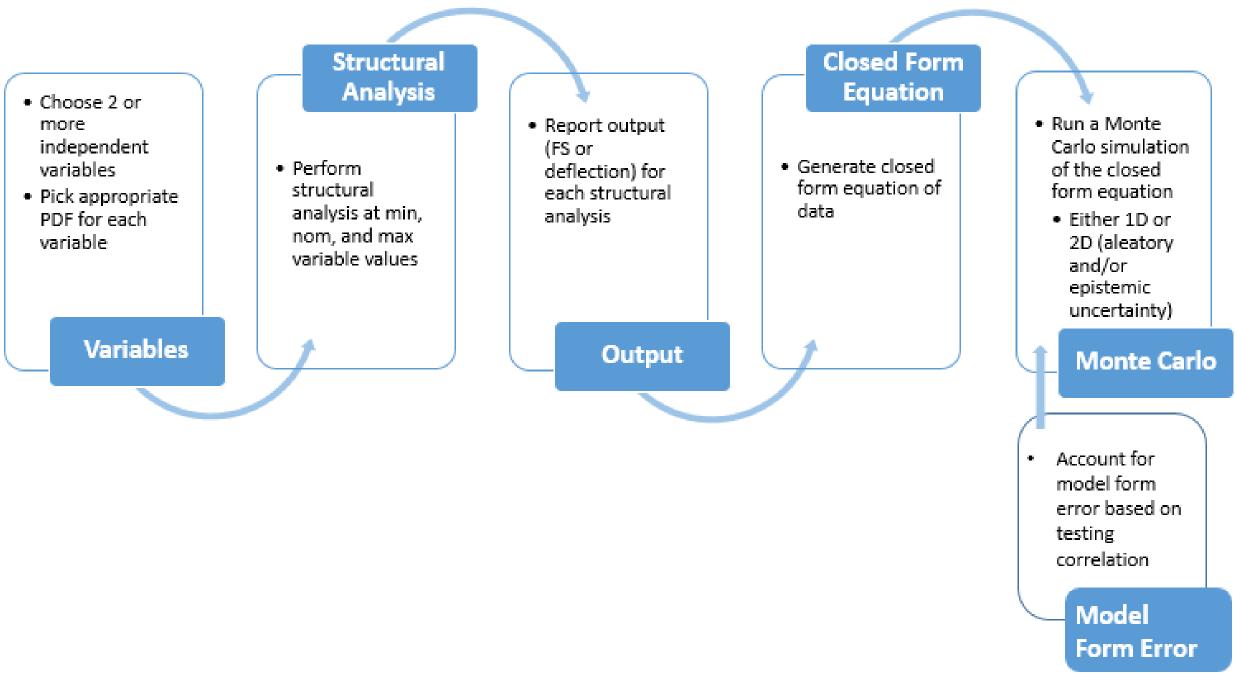

Figure 1 UQ Process Flow

II. UQ Variables

Choosing which variables to use for a UQ can be challenging for analyzing metal structures. The uncertainty in analysis variables isn't always evident, but the list below may be helpful in selecting which uncertainties can have a large effect on the results of an analysis. A UQ must contain at least two independent variables, further discussed in section III.

A. Loads

Loads are the primary driver of stress for a metals structure and are the source of the most variation for the aleatory unknowns of a structural analysis. There are usually multiple load sources affecting a structure simultaneously, which can vary in both directions and magnitude. Typically, a bounded load is established as the required load. However, loads will vary from a minimum to a maximum during a time bin or loading event. A probability distribution function (PDF) of the loads or how the loads are combined should be used during a UQ, assuming the distribution is available. A PDF is a function of a discrete variable such that the integral over any interval is the probability that the random variable specified by it lies within the interval. Structural loads include, but are not limited to, thermal, aero, vibro-acoustic, component load factors (CLFs), and attach loads.

B. Material Properties

Metal material properties and allowables often come from textbooks or other verified documents, where A-basis material properties were generated from PDF data [3]. While these values are usually taken as definite points, A-basis allowables are defined as 99% of the population with a 95% confidence interval is expected to be equal to or exceed the reported value, and thus have an inherent uncertainty. Often the specific material test data is not generally available, but good statistical approximations are easily attainable and should be used in the UQ. If A-basis material properties are not available, material testing can be completed to mitigate uncertainty. When non-linear, or plastic material properties are required (which should be the majority of the time when examining failure of primary structures), the API method can be used to generate the stress-strain curves. The API method is used to generate a curve of the plastic material properties of a metal from the input of a material's modulus, ultimate strength and yield strength. The curve is derived using the Ramberg-Osgood equation [4].

Due to its ease of generation, a PDF for the material properties of a metal structure is the most common variable used when looking at uncertainty. It is important to note that yield strength and ultimate strength of metal materials are often not independent variables, which is a requirement for a UQ. Furthermore, plastic, or nonlinear, material properties are often used to predict failure in primary metal structures. A true stress-strain curve from the PDF of the material properties can be generated for use to incorporate plastic material properties. A thermal knockdown factor (TDF) will still apply to these material properties based on requirements, unless the uncertainty of material temperature is evaluated.

C. Material Temperature

The temperature of a metallic material determines the TDF, which applies to both the yield and ultimate stress allowables of the component. The TDF lowers the ultimate and yield allowables of a metal based on the temperature during component use. Typically, the maximum material temperature is applied to the structure for a given load event in order to bound the material properties. However, temperatures vary from a minimum to a maximum during a time bin or loading event. A PDF of the thermal loads can be input into the structural analysis model. The effect of the bounded TDF of a metallic material is usually greater than the difference between a nominal and minimum value for an A-basis material. So, if a PDF for material temperature is available, it should be used in the analysis.

D. Geometry

Geometric dimensions of a component can vary between the drawing and machining tolerances. These dimensional uncertainties can affect the analysis model failure prediction and the margin at that location. A typical structural analysis is often built with nominal geometric dimensions. A PDF can be created from previously collected data of inspections of similar components or estimated based on history, and input into a structural analysis model. When the geometry tolerance has a significant impact on the margin of a structure, it should be included as a variable in the uncertainty analysis. However, typical drawing tolerances are often quite small with regards to the overall component size, especially for primary structures. Therefore, for most cases, the structure's margin often does not significantly vary from maximum to minimum material condition. One example where geometry variance is important is for membrane thickness of pressure vessels.

E. Finite Element (FE) Analysis Model Boundary Conditions

Model boundary conditions are an epistemic uncertainty in an FE analysis. The model geometry for the analysis can include neighboring structures that may introduce loads and stresses into the component being analyzed. By including the neighboring structures, the mathematical or simulated boundary conditions, which can be inaccurate, are moved further away from the structure of interest. This allows the uncertainty from the mathematical boundary condition to occur in a portion of the analysis model where failure is not being assessed. Also, the specific boundary condition applied to a structure can affect the outcome of the analysis.

Boundary conditions can be the biggest factor impacting FE analysis results. Boundary conditions should be considered closely in any structural analysis. Variables of boundary conditions that can be varied include, but are not limited to, type of contact, contact settings, convergence settings, type of mathematical boundary conditions and the geometry of non-critical structures that are included in the FE model.

The above list documents the typical variables that should be considered before starting for a standard UQ. Specific analyses may have other epistemic or aleatory variables that can affect the outcome of the analysis. If a variable affects the analysis results and a PDF is able to be generated for input, that variable should be used in the uncertainty analysis.

III. Process

This process provides a baseline strategy for completing a UQ for metal structures. Individual analyses may deviate from this process based on component specific problems, data available or the needs from the analysis. A UQ focuses on optimizing the result for the most probable failure mode(s) or maximum predicted deflection for a particular structure. The process below provides an in-depth discussion of the flowchart shown in Figure 1.

1) At least two independent variables (examples in section II) need be chosen to assess the UQ. An appropriate PDF should be decided upon for each variable based on data or estimates.
   Variables for the uncertainty analysis need to be independent on each other. A linear regression analysis should be performed to determine if the variables are independent if there is any question of their dependence.
   More than two variables can be used, which would reduce the uncertainty. However, more variables will create additional work and more time needed during the future process steps. The same variable type (material, loads, etc.) may be used for multiple variables. For example, the two variables could be both axial and radial loads for a structure. Or, for a failure mode that's dependent on two components, the two variables could be each of the component's material properties.

2) The structural analysis is then performed at the combination of minimum, nominal, and maximum values, according to the specific PDF, for each variable.
   For some analyses, this may only involve plugging in different numbers into spreadsheets or other hand calculations. However, for most major structures, the critical margins are calculated with an FE analysis model. Examining only 2 variables is recommended because FE model runs are typically on the order of hours, and require a minimum of 9 model runs. As more variables are examined, the time needed to complete the analysis exponentially increases, in both setup and model run time. If a spreadsheet is used to calculate the critical FS, a UQ with more than 2 variables is easier to complete.

3) The output of each run of the structural analysis is either the design capability of the structure, or the maximum predicted deflections.
   A hypothetical example of an acceptable result table format to report results is shown in Table 1. If more than 2 variables are examined, the table size would increase accordingly.

Table 1 Example of Table Format for Results of Analysis Runs

		Load		
		Min	Nom	Max
Stress-Strain Curve	Min A-basis	1.40	1.52	1.62
	Mean A-basis	1.57	1.70	1.83
	Max A-basis	1.70	1.83	1.92

   In the example table format above, the two variables varied are the material properties of the structure and load it sees. The output is the factor of safety (FS) of the structure. Any variable may be used in an uncertainty analysis and varied in a similar way.

4) Next, a closed form equation or surrogate model for design capabilities or predicted deflections is created. The equation should have a good fit ($R^2 > 95\%$) for the data, where R^2 is a statistical measure that represents the proportion of the variance of a dependent variable that is predictable from the independent variable(s).
   Any method of modeling the data is acceptable, assuming it is a good fit (R^2 value). One example is the use of a stepwise regression method. For a 3x3 table of variables, the R^2 value should be higher, likely greater than 99%.

5) A Monte Carlo simulation of the closed form equation or surrogate model is then performed. The model should be run for enough iterations such that there is no change in output. The Monte Carlo simulation can either be 1-dimensional (only looking at either aleatory or epistemic uncertainty), or 2-dimensional (looking at both aleatory and epistemic uncertainty).

   The results of the Monte Carlo simulation of design capability will show a reliability based SF for the structure. The result of a simulation of deformations can show an upper 3-sigma estimate of the deflection in question.

   A typical Monte Carlo simulation of a 3x3 table of variables should be performed at least 10,000 times. However, as long as the output is not changing with subsequent simulations, the exact number of simulations may change depending on the variables and inputs to the uncertainty analysis.

6) The analysis model should be correlated to structural test results to find the percent error between the experimental structural test and predicted results. If there is significant model form error, the statistical distribution of this epistemic error needs to be accounted for during the Monte Carlo simulation.

   If the percent error is negligible (test result within a percent or two of the predicted result), model form error can be ignored in the UQ process, as it the error is likely in the noise of the FE model. For example, predictions of tensile failure modes in pressurized metallic membranes are typically easy to predict, and thus do not require an assessment of model form error.

   For a 1D analysis of an aerospace structure, the results should be reported based on a 99.87% probability of not exceeding the given FS or maximum deflection (assuming modeling uncertainties are negligible). For a 2D analysis, results should be reported based on 99.87% reliability and 95% credibility [2].

IV. Example

Let's look more in-depth at the hypothetical example from Table 1. In that example, the two independent variables chosen for uncertainty evaluation were the axial load applied to the structure, and the A-basis stress-strain curve for the metal material of the structure. It was assumed that each variable had a normal distribution for its PDF. The maximum and minimum values of both the axial load and the material properties were given as the +/-3σ values of the PDFs.

After the variables and their associated PDFs were determined, nine FE models were run, for all combinations (maximum, nominal, and minimum) of both the axial load applied and the stress-strain curve for the material. The models were post-processed, looking at the failure mode of the structure. The individual FS from each model were placed into Table 1 based on the initial variable condition.

Next, a closed form equation, or surrogate model was created based on the data in Table 1. A stepwise regression method was used to model the FS data. The resulting surrogate model, shown in equation 2, was shown to have a good fit to the input data with an R^2 value of 99.6%.

$$FS = 1.70 + 0.03889 * Axial\ Load + 0.05056 * StressStrain - 0.003889 * StressStrain\hat{}2 \tag{2}$$

This closed form equation was Monte Carlo simulated 100,000 times. For this simulation, model form error was assumed to be negligible. Ideally, there should be structural test data to back up that assumption. The result of that simulation is shown in Figure 2. The plot has the observed FS on the x-axis and the expected normal value, in terms of sigma away from the mean, on the y-axis

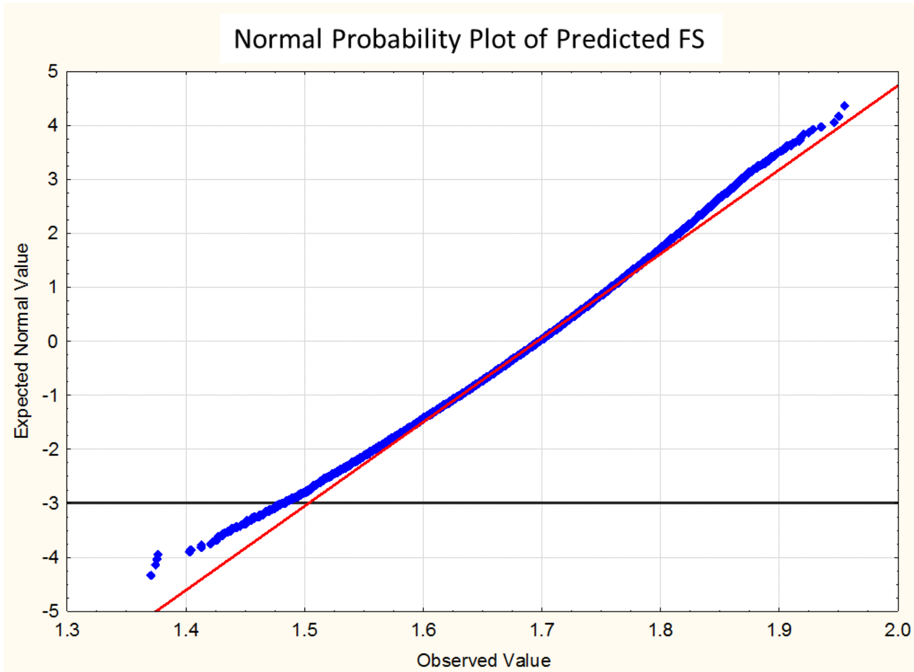

Figure 2 Monte Carlo Simulated Results for Example

The Monte Carlo simulation and subsequent plot show a few key pieces of data. The lower three-sigma estimate for this UQ was a FS of 1.48. The lowest FS shown from the 100,000 runs was 1.37, while there was only a 1 in 25,000 chance of having a FS below 1.40. If the required SF for this structure was 1.40, this UQ analysis creates a lot of confidence in the capability of the design of the example structure. Furthermore, if this hypothetical structure was large, and thus mass-critical, the UQ shows that there is room for optimization, as the lower three-sigma FS was above the 1.40 limit.

V. Conclusion

In a UQ, the importance of testing cannot be understated. In order to perform a successful UQ, the failure modes of a given structure must be well understood. The testing of such structures also allows the analyst to correlate the test data to the model, quantifying the epistemic uncertainty in said analysis model. It is recommended that at least three qualification level structural tests are completed on any structure for which a UQ is used as the primary method of analysis. However, it is not always necessary to test the full structure. Subscale testing can have the same desired effect as full-scale testing in the correct situation.

While a UQ for a metal structure can be useful, it is not a one size fits all analysis solution. For most analysis of smaller components, the mass-savings is negligible compared to the time and effort needed to complete a UQ. Furthermore, the necessity or usefulness of a UQ may not be as pronounced in industries outside of aerospace, where reducing the mass of large structures isn't as critical. For example in the aerospace industry, for a constant thrust, the lighter the launch vehicle is, the more payload said vehicle can carry. Reducing the uncertainty in a structural analysis to reduce the mass of the structure makes UQ such a useful tool in the aerospace industry.

References

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