



Uncertainty Quantification of Metal Structures

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Standard Metal Structure Analysis

- Conservative approach
- Bound aleatory uncertainty
 - Uncertainty due to inherent variation or randomness
 - “Known unknowns”
- Apply a safety factor (SF) to account for epistemic uncertainty
 - Uncertainty due to lack of knowledge
 - “Unknown unknowns”
- Margin calculation for a failure mode:

$$\text{Margin} = \frac{\text{Allowable}}{SF(\text{Model Result})} - 1$$

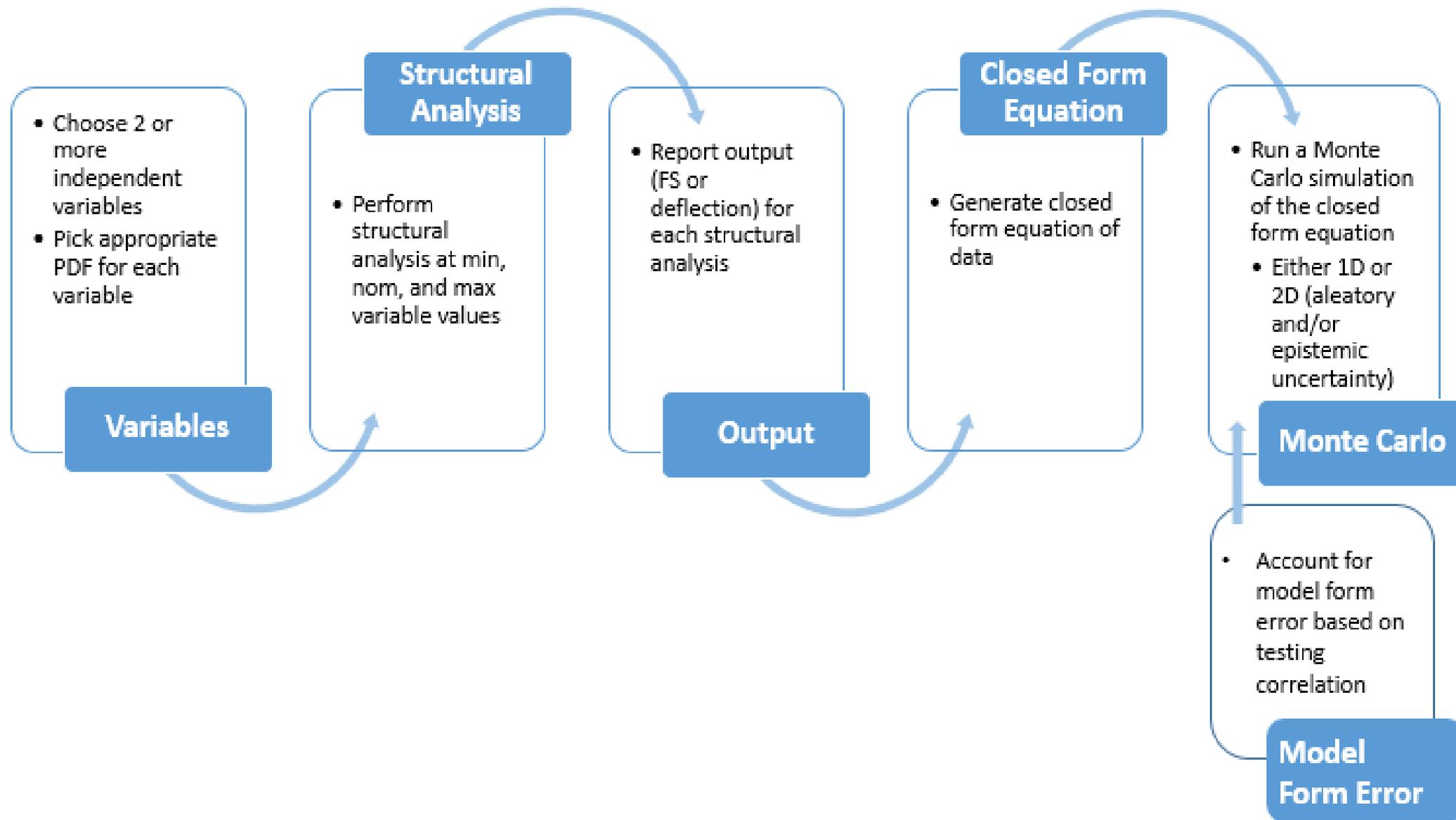
Uncertainty Quantification (UQ)

- Assess the uncertainty of aleatory and epistemic variables
- Creates a reliability based SF
- Metals do not have as much inherent uncertainty as other disciplines in aerospace engineering
- Will discuss variables and process for performing UQ analysis on a metal structure
 - Can result in significant mass saving for primary structures

UQ Variables

- Loads
- Material properties
- Material temperature
- Geometry dimensions
- Finite element (FE) analysis model boundary conditions
- Any analysis input with a probability distribution function (PDF)

UQ Process Overview



Example UQ: Step 1

- Hypothetical metal structure UQ analysis to determine FS
 - Assuming goal FS of 1.4
- Step 1: Variables
- Two independent variables chosen
 - Axial load applied to structure
 - A-basis stress-strain curve of metal material
- Normal distribution PDF selected for each variable

Example UQ: Steps 2 & 3

- Step 2: Structural analysis performed at minimum, nominal, and maximum variables
 - Upper and lower 3σ values for max/min
- Step 3: Report FS output from FE model

		Load		
		Min	Nom	Max
Stress-Strain Curve	Min A-basis	1.40	1.52	1.62
	Mean A-basis	1.57	1.70	1.83
	Max A-basis	1.70	1.83	1.92

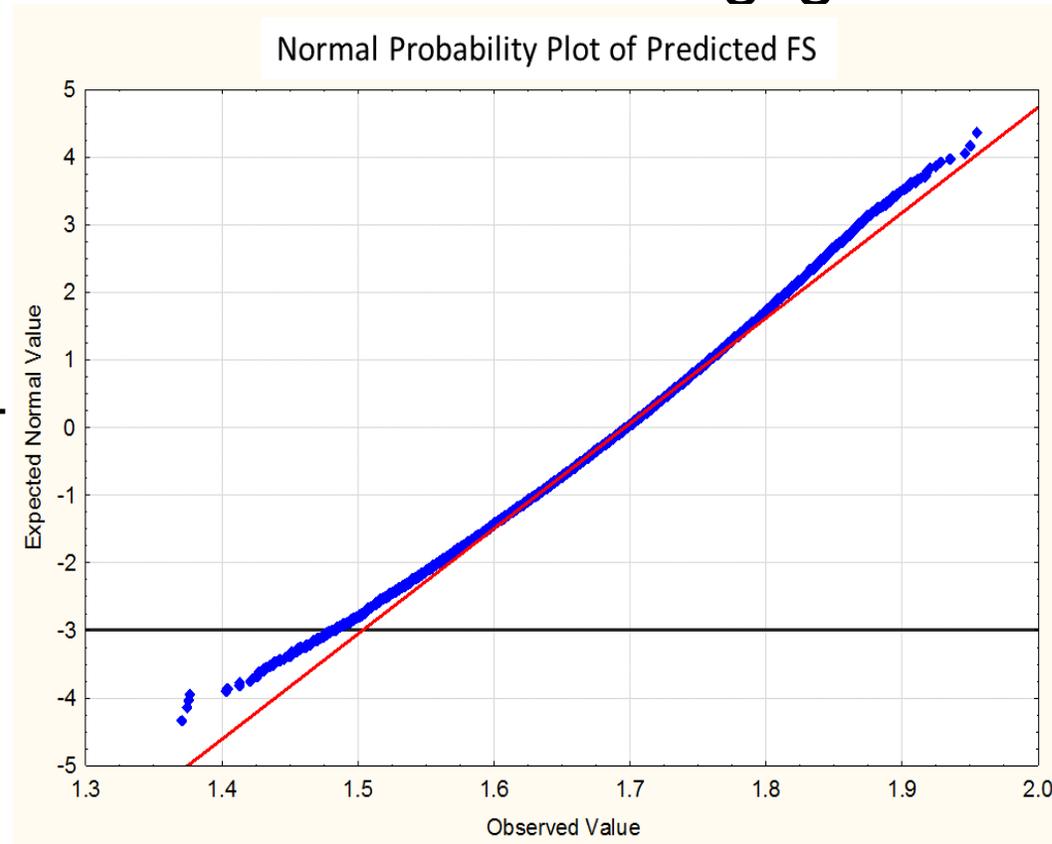
Example UQ: Step 4

- Step 4: Closed form equation (surrogate model) created from FS data
- Stepwise regression method was used for example
 - Good fit, $R^2 = 99.6\%$
- Surrogate model for UQ:

$$FS = 1.70 + 0.03889 * Axial Load + 0.05056 * StressStrain - 0.003889 * StressStrain^2$$

Example UQ: Steps 5 & 6

- Step 5: Monte Carlo simulated 100,000 times
- Step 6: Model form error was assumed to be negligible for example
- Monte Carlo results:
 - Lower 3σ FS = 1.48
 - Lowest FS = 1.37
 - 1 in 25,000 chance $FS < 1.4$



Conclusion

- UQ is not a one size fits all solution
- Most useful on large, primary structures
 - For aerospace applications, mass-savings is an important design driver
- Structural testing is critical
 - Need to fully understand failure modes of the structure
 - Allows correlation of test data to FE model
 - Model form error should be used in Monte Carlo simulation



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